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AN OPTIMAL DESIGN OF SIMPLE SYMMETRIC
LAMINATES UNDER THE FIRST PLY FAILURE
CRITERIA

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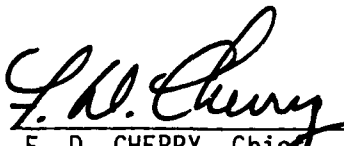S   H

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20. ABSTRACT *(Continue on reverse side If necessary and Identify by block number)*

Optimal design of various types of symmetric laminates for T300/5208 graphite epoxy composites was investigated under the first ply failure criteria. The symmetric laminates considered include continuous laminate and angle ply laminate. The optimal design angles ϕ were obtained and presented in graphic form as functions of the loading conditions (N_1, N_2, N_6). The results presented here are directly useful for designers for making a choice of composites for optimal performance.

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FOREWORD

This report was prepared in the Mechanics and Surface Interactions Branch (AFWAL/MLBM), Nonmetallic Material Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio. The work performed under Contract F33615-79-C-5129, Project UES 718.

The work reported herein was performed during the period October 1980 to September 1981. Dr. Won J. Park is a Senior Scientist from Universal Energy Systems, Inc. and Professor of Mathematics and Statistics at Wright State University.

The author wishes to express his sincere gratitude to Dr. Stephen W. Tsai for his encouragement and guidance in the course of this work.

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SECTION 1

INTRODUCTION

One of the fundamental and frequently recurring optimization problems in fiber composite structure is the design of laminates subject to various inplane loading conditions, considering strength and stiffness. It is well understood that, in laminate designs, the most important design variables are ply orientation angle, ply thickness and volume fraction of fibers.

Kicher and Chao [1] has considered an optimal laminate design problem, in which the ply thickness of preassign ply angle was considered as the only design variable under the minimum weight optimization criteria. Under this criteria the volume fraction of fiber is indirectly related to the ply thickness, but ply orientation angle is completely independent in the design and causes a serious difficulty in laminate optimization design.

A new approach in the design of laminates was proposed in this report, which utilizes a first ply failure (FPF) criteria as the objective function in the design. Under the FPF criteria, the ply orientation angle is a very sensitive design variable and the optimal laminate designed has the most strength in the sense of first ply failure criteria.

The design of various simple symmetric laminates (see Figure 2 for the descriptions of laminates A-F) of the composite material T300/5208 was carried out under the FPF criteria. The fraction of fiber content is constant and each ply has equal thickness throughout the laminate.

The design angles ϕ of the laminate A-F are obtained under various loading conditions (N_1, N_2, N_6) and are presented in Figures 18-29. In addition, the modulus, compliance and engineering constants of the laminate A-F are tabulated in terms of the ply design angle ϕ and are presented in Figures 3-17.

It is interesting to compare the optimal design angle ϕ with the corresponding modulus, compliance and engineering constants for these laminates. It is also noted that the FPF optimal criteria can be applied to the design of more general laminates but this requires very sophisticated computer optimization techniques.

SECTION II

INPLANE STIFFNESS

The composite laminates, having a symmetry of stacking sequence about the midplane surface, behave as a homogeneous anisotropic plate. The effective modulus of the composite laminates is simply the arithmetic average of the modulus of the constituent plies. The main stress strain relations for a composite laminate are, for modulus

$$N_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + A_{16}\varepsilon_6$$
$$N_2 = A_{21}\varepsilon_1 + A_{22}\varepsilon_2 + A_{26}\varepsilon_6 \qquad (1)$$
$$N_6 = A_{61}\varepsilon_1 + A_{62}\varepsilon_2 + A_{66}\varepsilon_6$$

and for compliance

$$\varepsilon_1 = a_{11}N_1 + a_{12}N_2 + a_{16}N_6$$
$$\varepsilon_2 = a_{21}N_1 + a_{22}N_2 + a_{26}N_6 \qquad (2)$$
$$\varepsilon_6 = a_{61}N_1 + a_{62}N_2 + a_{66}N_6$$

where N_1, N_2 and N_6 are stress resultants over the thickness h of the laminate and are defined by

$$N_i = \int_{-h/2}^{h/2} \sigma_i \, dz \, , \quad (i = 1, 2, 6) \qquad (3)$$

and ε_i (i = 1, 2, 6) are inplane strain components, constant through the thickness. The modulus components A_{ij} are given in the following table:

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	1	U_2	U_3
A_{11}/h	U_1	V_1^*	V_2^*
A_{22}/h	U_1	$-V_1^*$	V_2^*
A_{12}/h	U_4		$-V_2^*$
A_{66}/h	U_5		$-V_2^*$
A_{16}/h		$\tfrac{1}{2}V_3^*$	V_4^*
A_{26}/h		$\tfrac{1}{2}V_3^*$	$-V_4^*$

In which U_i (i=1, 2,...,5) are invariances given in Equation (3.15) of Tsai and Hahn [2] and

$$V^*_{(1,2,3,4)} = \frac{1}{h} \int_{-h/2}^{h/2} (\cos 2\theta, \cos 4\theta, \sin 2\theta, \sin 4\theta)\,dz \qquad (4)$$

The engineering constants for the laminate are defined as follows:

$$E_1^0 = 1/ha_{11}$$
$$E_6^0 = 1/ha_{66} \qquad (5)$$
$$\nu_{21}^0 = -a_{21}/a_{11} \quad .$$

The equation of approximate first ply failure surface (strain envelop) is given by

$$\varepsilon_1^2 + \varepsilon_2^2 + \tfrac{1}{2}\varepsilon_6^2 = 2b^2$$

(see Equation (7.102) of Tsai and Hahn [2]. Let us set

$$Q = \varepsilon_1^2 + \varepsilon_2^2 + \tfrac{1}{2}\varepsilon_6^2 \quad , \tag{6}$$

then FPF optimal criteria is to minimize the value of Q. The objective function Q has another meaning, mainly the value $\sqrt{Q}$ is the norm or length of the strain vector $(\varepsilon_1, \varepsilon_2, \varepsilon_6/\sqrt{2})$. Hence, the FPF criteria minimizing the value of Q is the equivalent to minimizing the norm of strain vector $(\varepsilon_1, \varepsilon_2, \varepsilon_6/\sqrt{2})$. The laminates that we have considered to design in this report are so simple that Q is a function of the design angle ϕ only for given loading conditions (N_1, N_2, N_6).

SECTION III

CONTINUOUS LAMINATE

It is interesting to consider a symmetric laminate whose ply orientation angle increases uniformly. We call this laminate a continuous laminate. If the ply orientation angle sweeps between $-\phi$ and ϕ, then ϕ is called the sweeping angle of a continuous laminate. The relation between laminate depth Z and ply angle θ is given by (See Figure 1),

$$Z = \left(\frac{h}{4}\right) \left[\left(\frac{\theta}{\phi}\right) + 1 \right], \quad -\phi \leq \theta \leq \phi \tag{7}$$

Note that a continuous laminate with one complete revolution of plies has the sweeping angle $\phi = 90°$. The modulus and compliance of the continuous laminate for $0° \leq \phi \leq 90°$ are given in Figure 8 and 14.

In FPF optimal design of the continuous laminate, the sweeping angle ϕ is the only design variable for given loading conditions (N_1, N_2, N_6).

SECTION IV

DESIGN OF LAMINATES

In addition to the continuous laminate, various laminates such as angle ply laminate and angle ply laminate with additional plies of 0, 90, -45 or 45 degree orientations were considered for FPF optimal design. For simplicity, these laminates are named Laminate A, B,....,F, which are specified in Figure 2. The composite material that is used is T300/5208 and its invariants values (in GPa) are;

$$U_1 = 76.37 \qquad U_2 = 85.73 \qquad U_3 = 19.71$$
$$U_4 = 22.61 \qquad U_5 = 26.88 \; .$$

Table 2 gives the expression of V_i^* (i = 1, 2, 3, 4), where $V_i^* = V_i/h$, for the Laminate A-F in terms of design angle ϕ.

It is clearly seen from Equation (1), (2) and (6) and Tables 1 and 2 that $Q = \epsilon_1^2 + \epsilon_2^2 + \frac{1}{2}\epsilon_6^2$ is a function of ϕ and (N_1, N_2, N_6), and hence Q depends only on ϕ if (N_1, N_2, N_6) are known values. Therefore under a given stress loading (N_1, N_2, N_6) the optimization problem is to find the value ϕ^* so that $Q = Q(\phi^*)$ is the minimum among all values of $Q = Q(\phi)$ for $0 \le \phi \le 90°$.

The stresses N_1, N_2, N_6 are normalized by the largest value so that $0 \le N_i \le 1$, i = 1, 2, 6. Accordingly, the laminate thickness h is set to be 1. The results of optimal design angle ϕ for lmainates A-F were presented in Figures 18-29. The cases that $N_1 = 1$ and $N_6 = 1$ are considered for each lminate. The case that $N_2 = 1$ is identical with that of $N_1 = 1$ if the laminate is rotated by 90 degrees.

Table 2   Computation of V*

Laminate	Ply Orientation Angle	V_1^*	V_2^*
A	$-\phi, \phi$	$\cos(2\phi)$	$\cos(4\phi)$
B	$-\phi, 0, \phi$	$\frac{1}{3}[2\cos(2\phi)+1]$	$\frac{1}{3}[2\cos(4\phi)+1]$
C	$-\phi, 90, \phi$	$\frac{1}{3}[2\cos(2\phi)-1]$	$\frac{1}{3}[2\cos(4\phi)+1]$
D	$-\phi, 0, 90, \phi$	$\frac{1}{2}\cos(2\phi)$	$\frac{1}{2}[\cos(4\phi)+1]$
E	$-\phi, -45, 45, \phi$	$\frac{1}{2}\cos(2\phi)$	$\frac{1}{2}[\cos(4\phi)-1]$
F	Continuous Laminate	$\frac{1}{2\phi}\sin(2\phi)$	$\frac{1}{4\phi}\sin(4\phi)$
$V_3^* = 0$, $V_4^* = 0$			

The comparison of Q values among the laminates A–F is given in Figure 30 and 31. The smaller the Q value the stronger the laminate in FPF criteria. When we apply our results of design angle ϕ in actual laminate design, the thickness of laminate h should be adjusted to meet the original values of stress conditions (N_1, N_2, N_6). The computations were carried out by the computer CDC 660 at Wright-Patterson Air Force Base.

SECTION V
CONCLUSIONS

The first ply failure criteria is proposed in the composite laminate design. It is found that design angle ϕ is very sensitive under this criteria and the results of design angle ϕ can be directly useful to laminate designers. Among the laminate A-F, laminate F performs overall the best in FPF criteria, and then followed by laminate B under maximum longitudinal stress and laminate E under maximum shear stress.

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1. T. P. Kicher and T. L. Chao, "Minimum Weight Design of Stiffened Fiber Composite Cylinders," Journal of Aircraft, Vol. 8, No. 7, July 1971, pp. 562-568.

2. S. W. Tsai and H. T. Hahn, Introduction to Composite Materials, Technonic Publishing Co., Westport, CT 06880, July 1980.
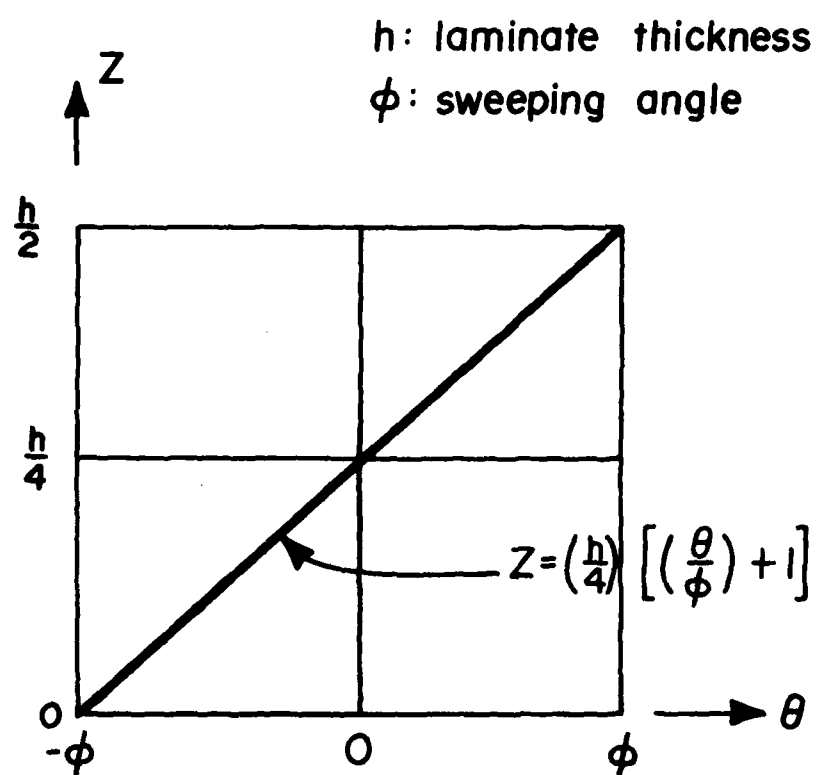
FIGURE 1   CONTINUOUS LAMINATE

1. LAMINATE A: $\{-\phi,\phi\}$
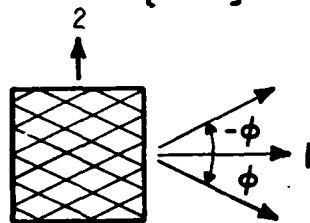
2. LAMINATE B: $\{-\phi,0,\phi\}$
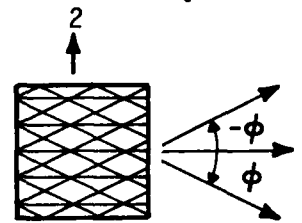
3. LAMINATE C: $\{-\phi,90,\phi\}$
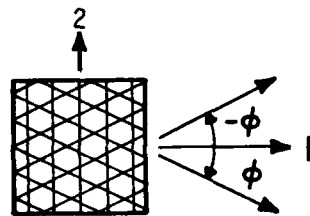
4. LAMINATE D: $\{-\phi,0,90,\phi\}$
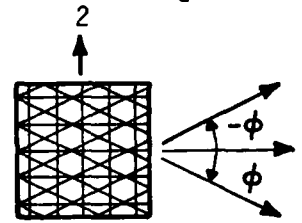
5. LAMINATE E: $\{-\phi,-45,45,\phi\}$
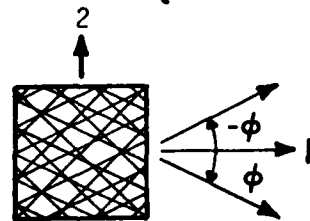
6. LAMINATE: F CONTINUOUS LAMINATE
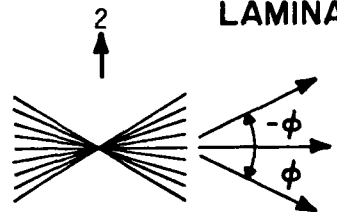
FIGURE 2. LAMINATES A – F

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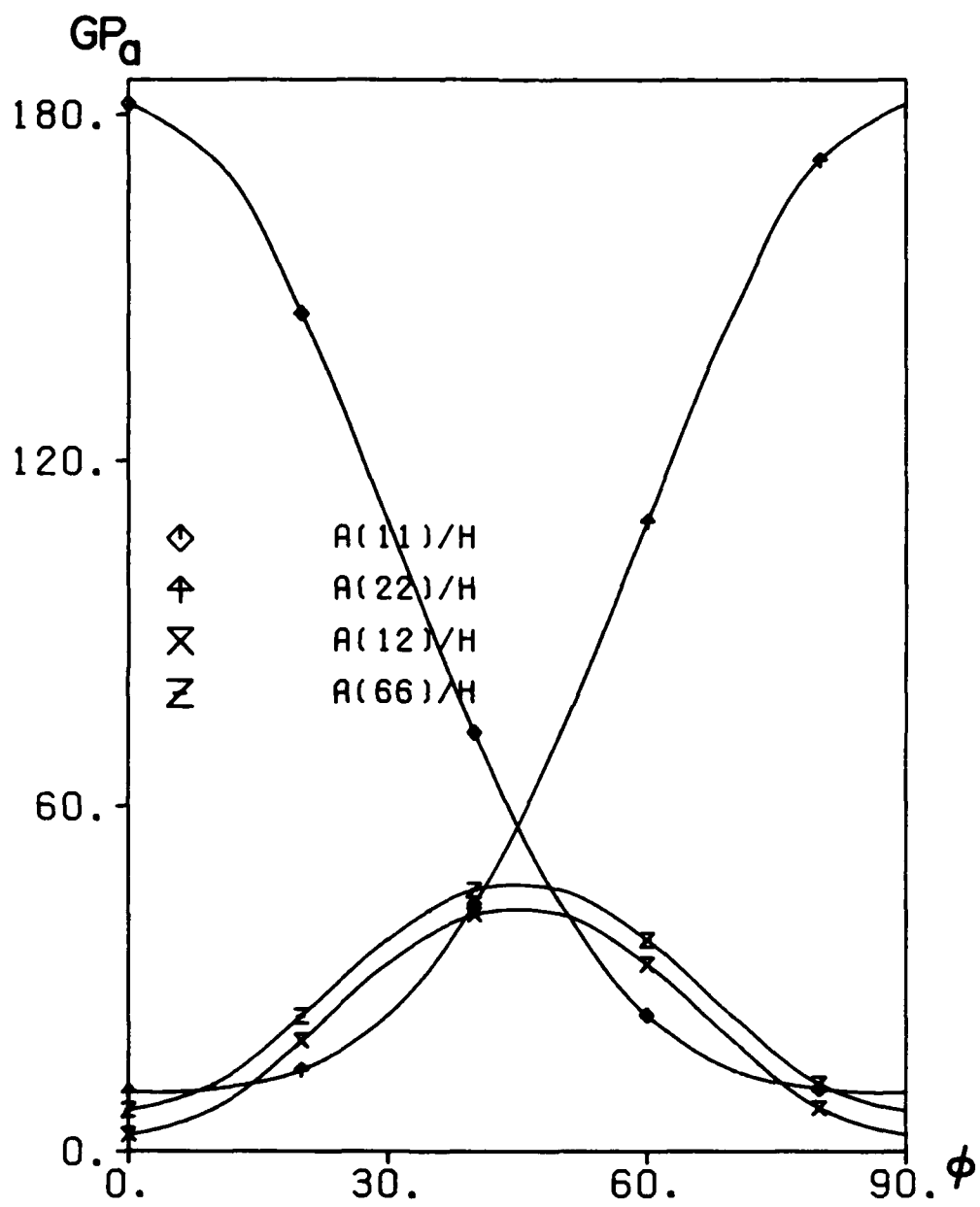
FIGURE 3 MODULUS OF LAMINATE A

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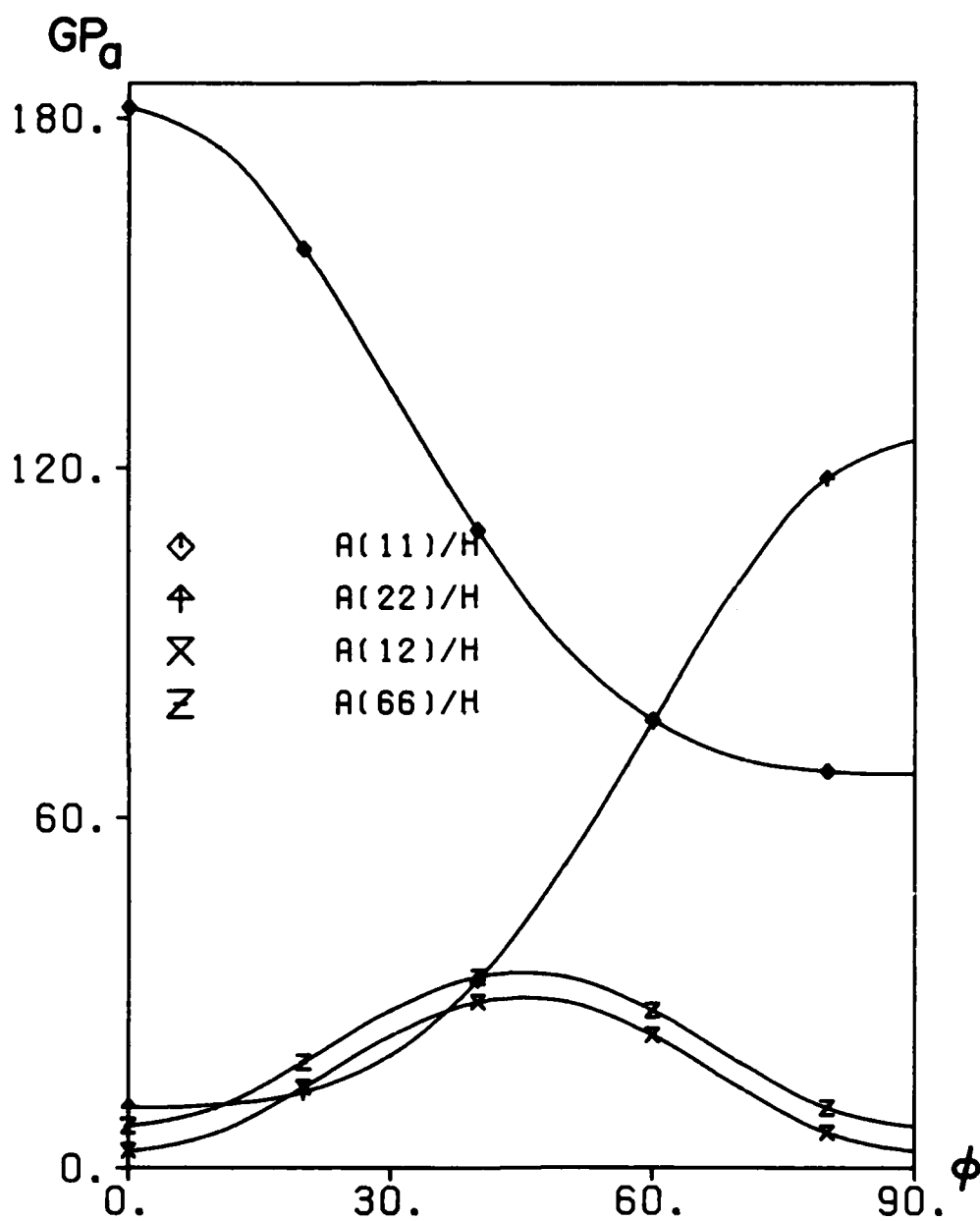
FIGURE 4 MODULUS OF LAMINATE B

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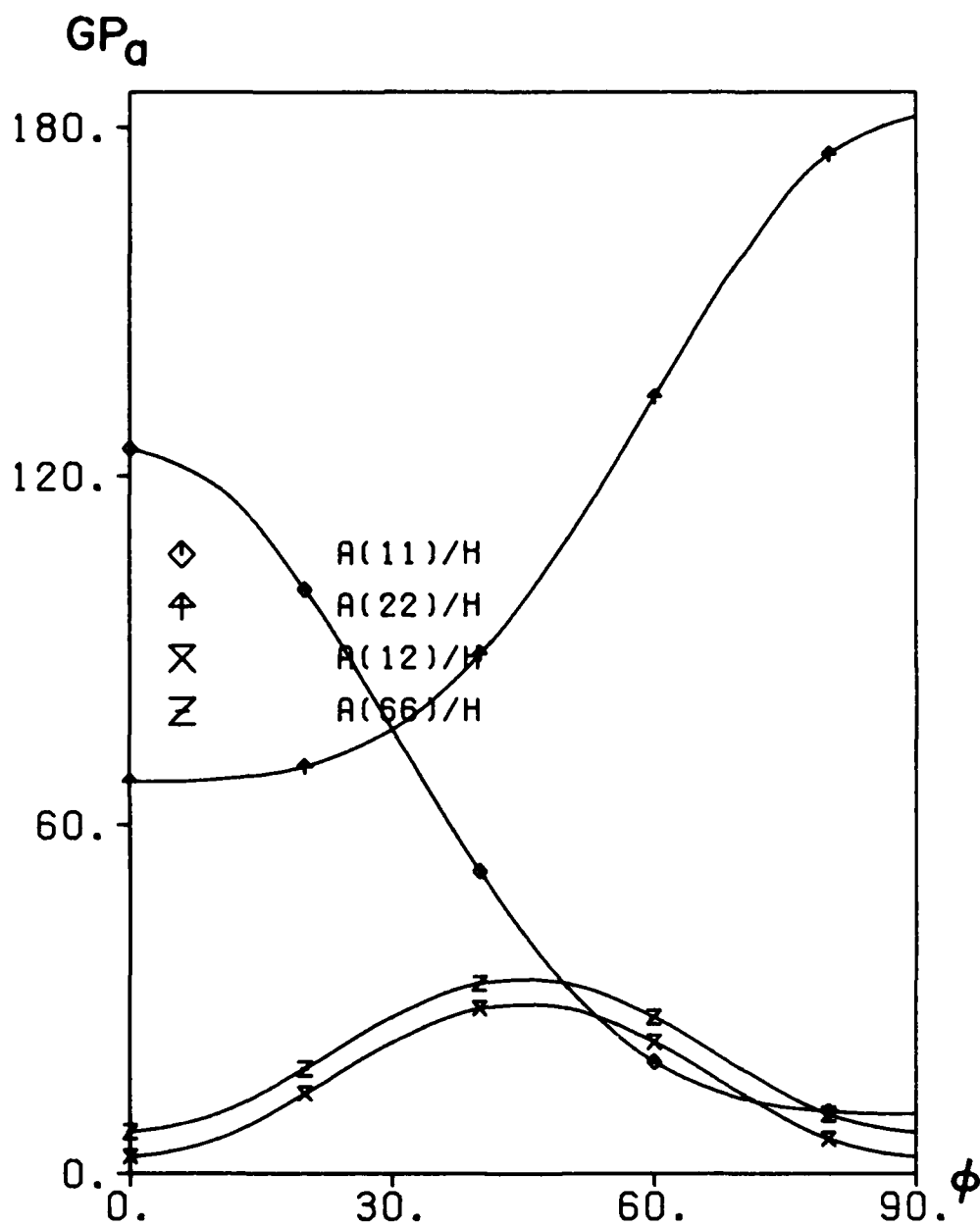
FIGURE 5 MODULUS OF LAMINATE C
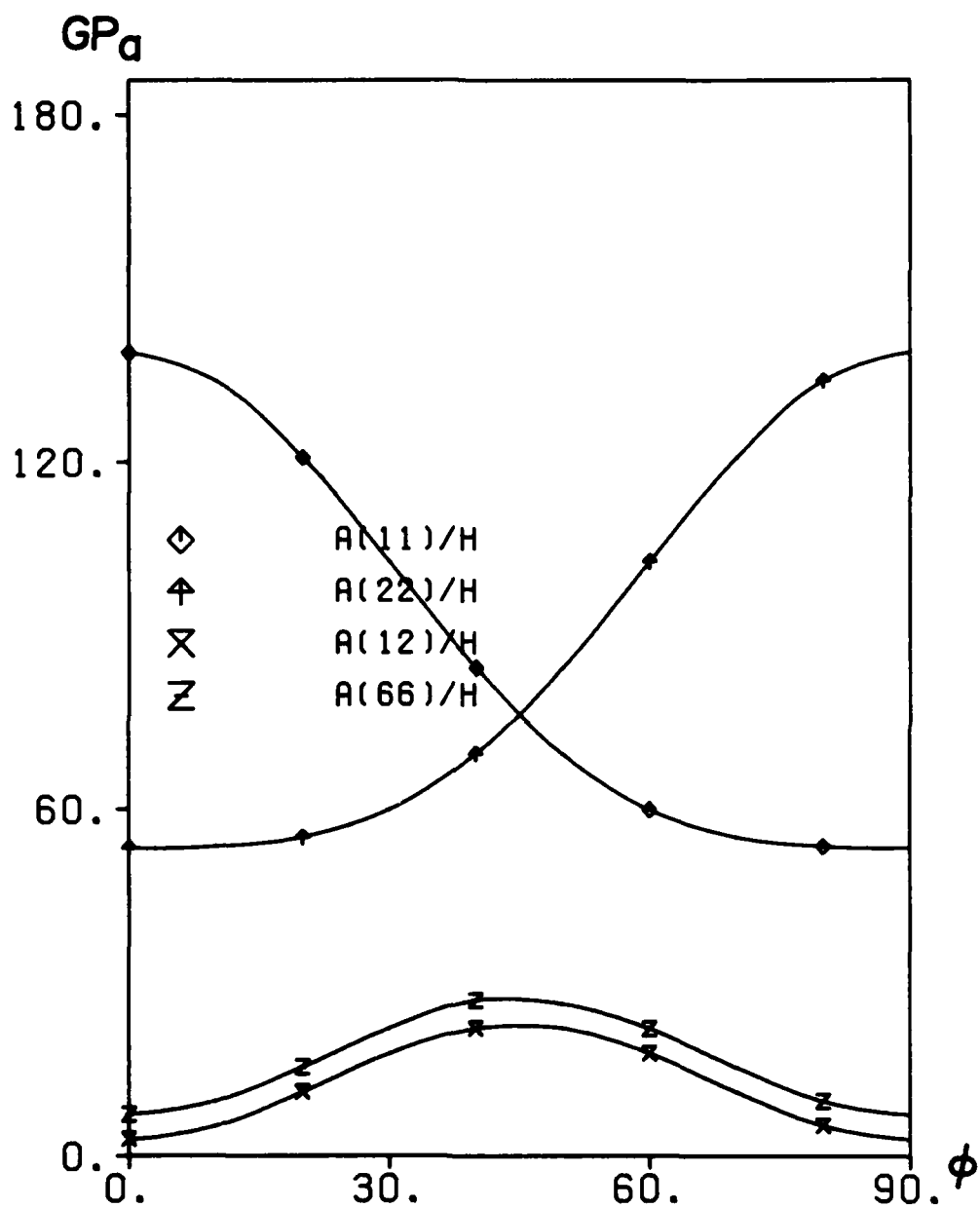
FIGURE 6 MODULUS OF LAMINATE D
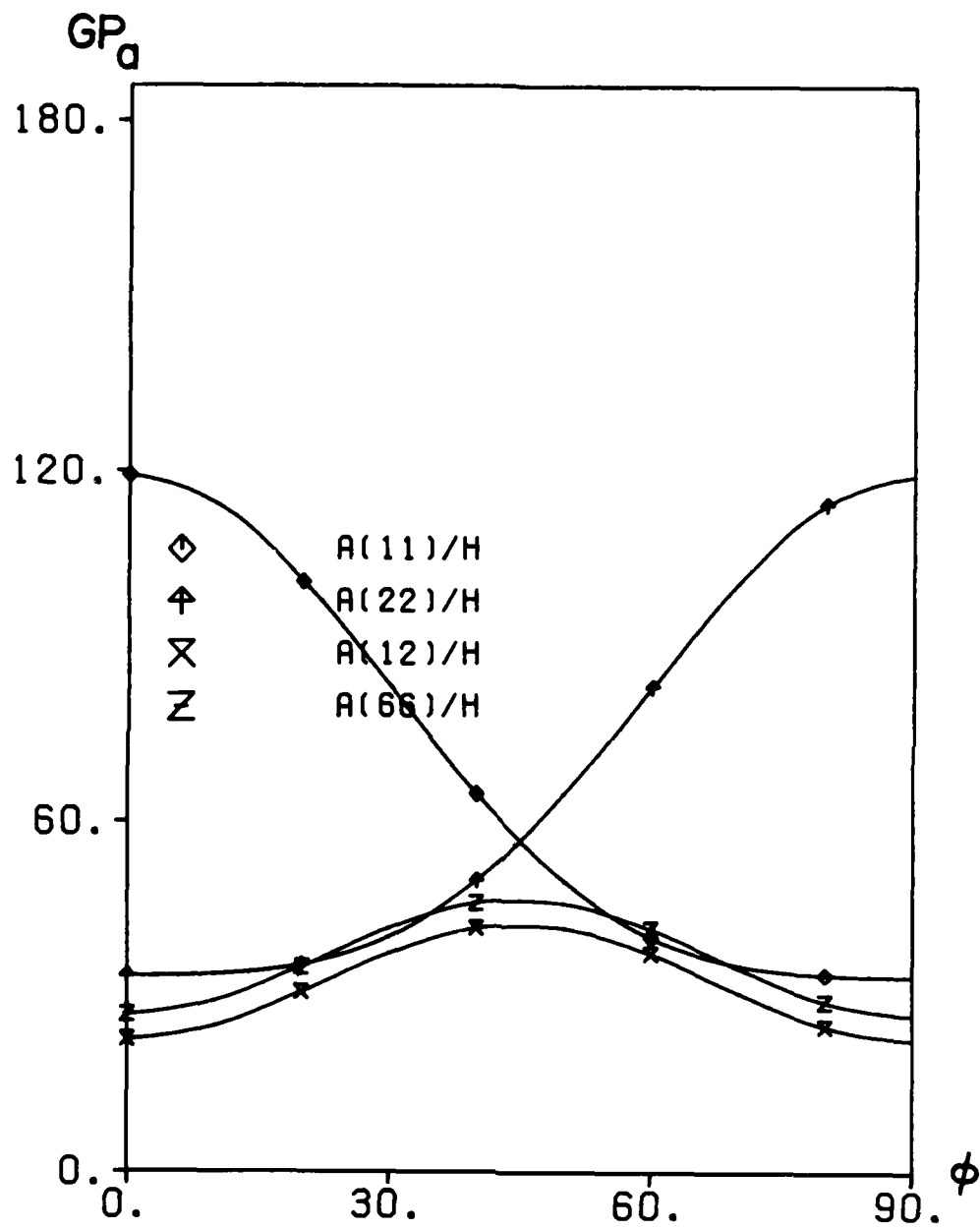
FIGURE 7 MODULUS OF LAMINATE E
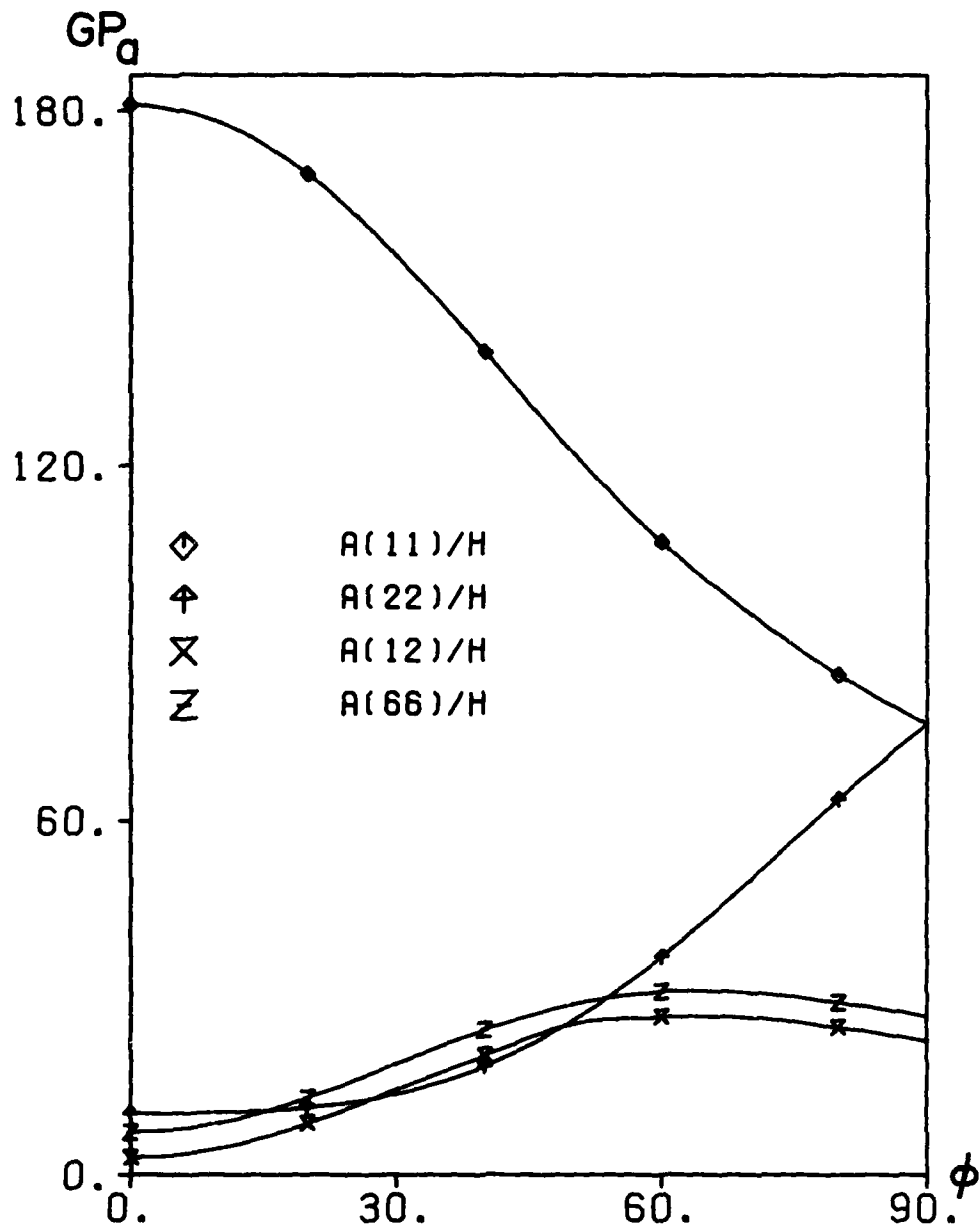
FIGURE 8 MODULUS OF LAMINATE F

$(TP_a)^{-1}$
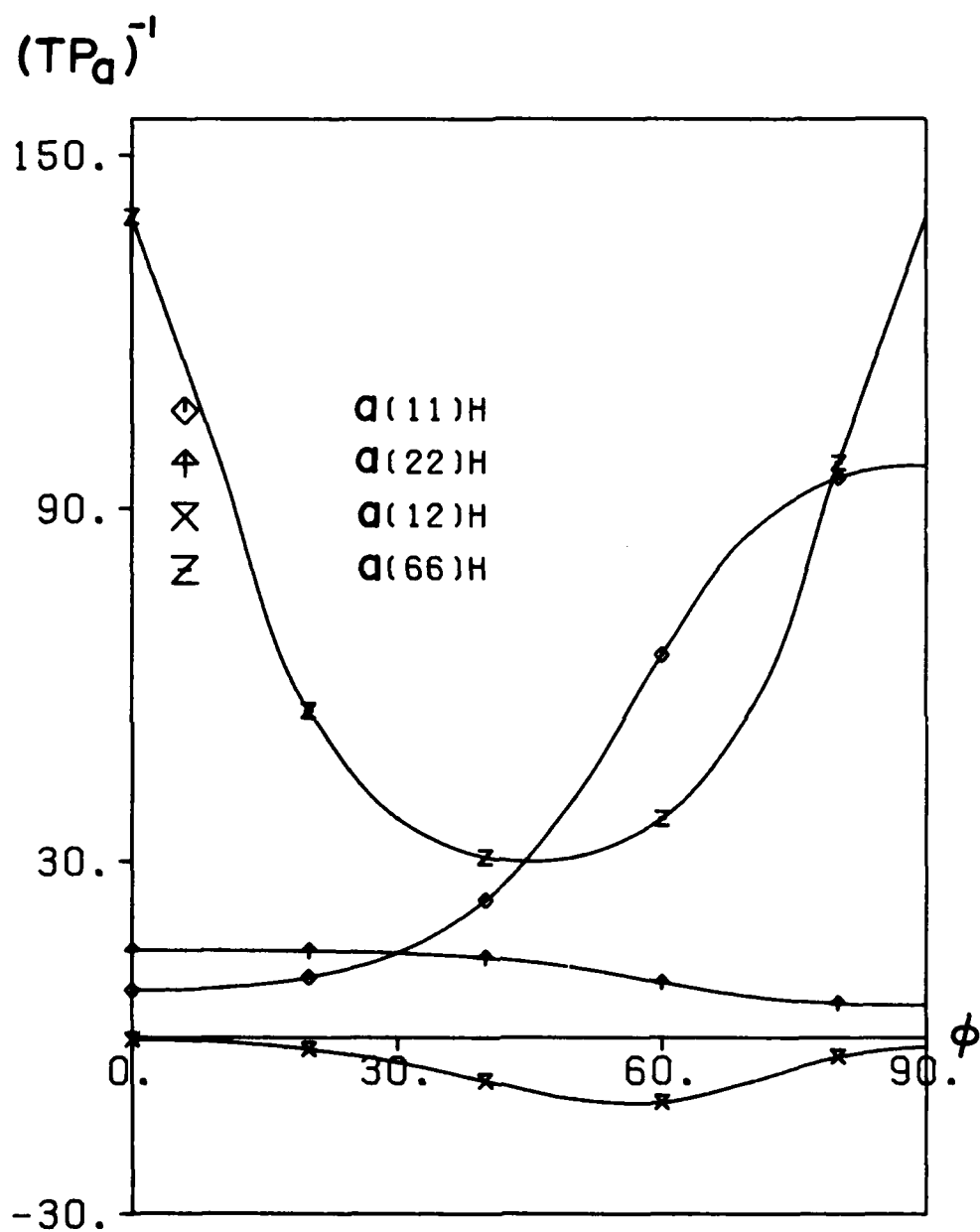
FIGURE 9 COMPLIANCE OF LAMINATE A

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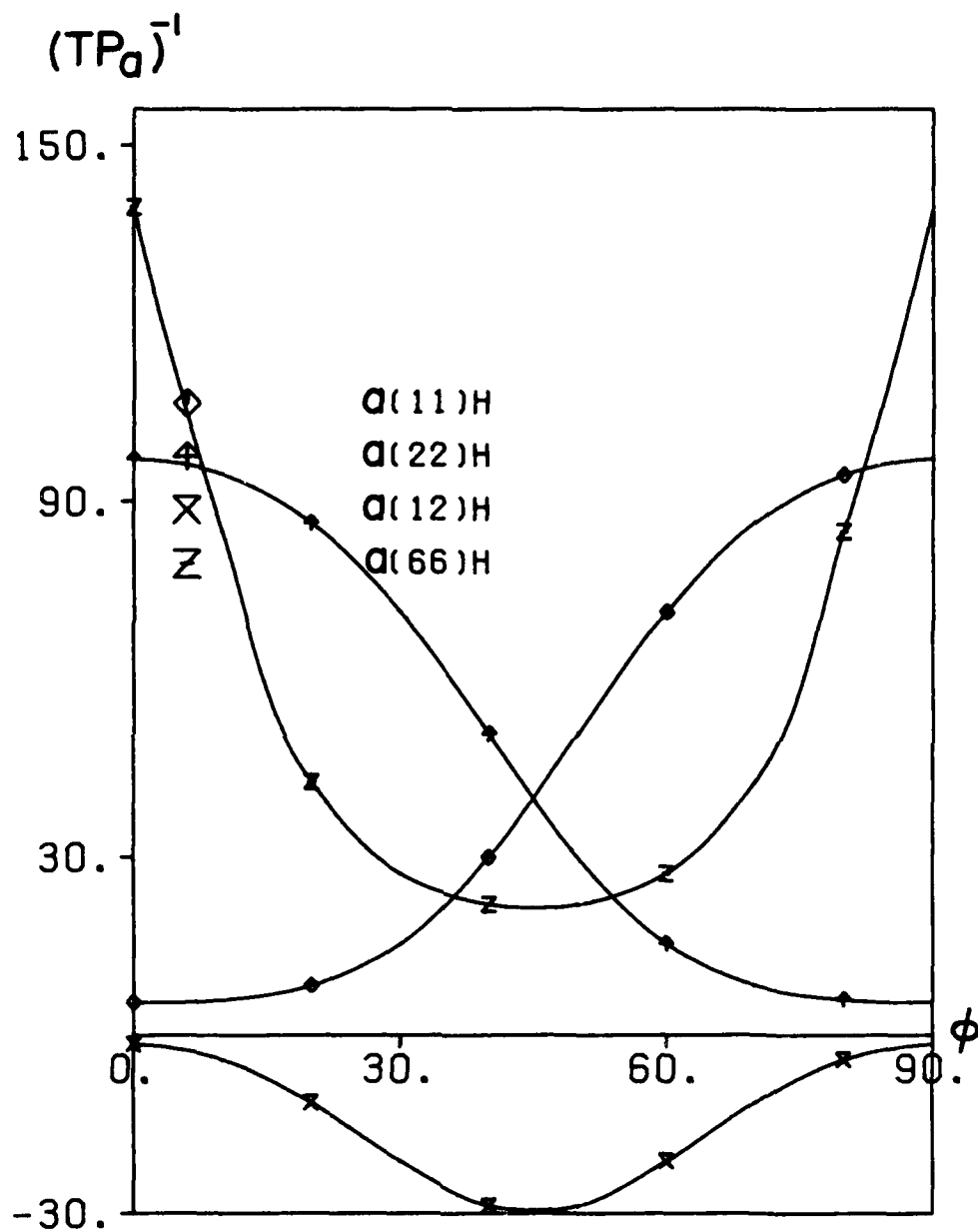
FIGURE 10 COMPLIANCE OF LAMINATE B

$(TP_a)^{-1}$

150.

90.

30.

-30.

◇ 𝖆(11)H
𝚫 𝖆(22)H
X 𝖆(12)H
Z 𝖆(66)H

0.   30.   60.   90. ϕ
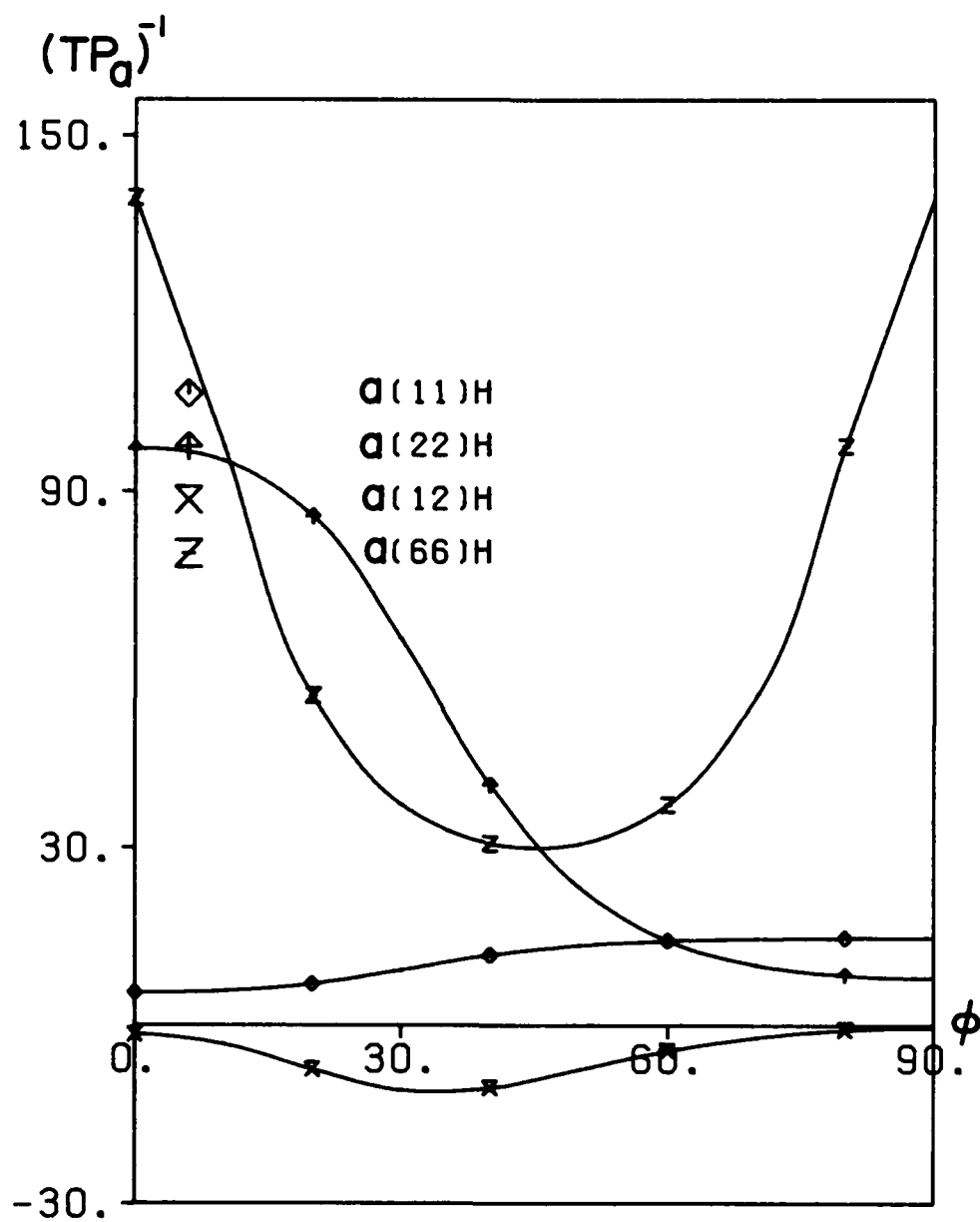
FIGURE II COMPLIANCE OF LAMINATE C

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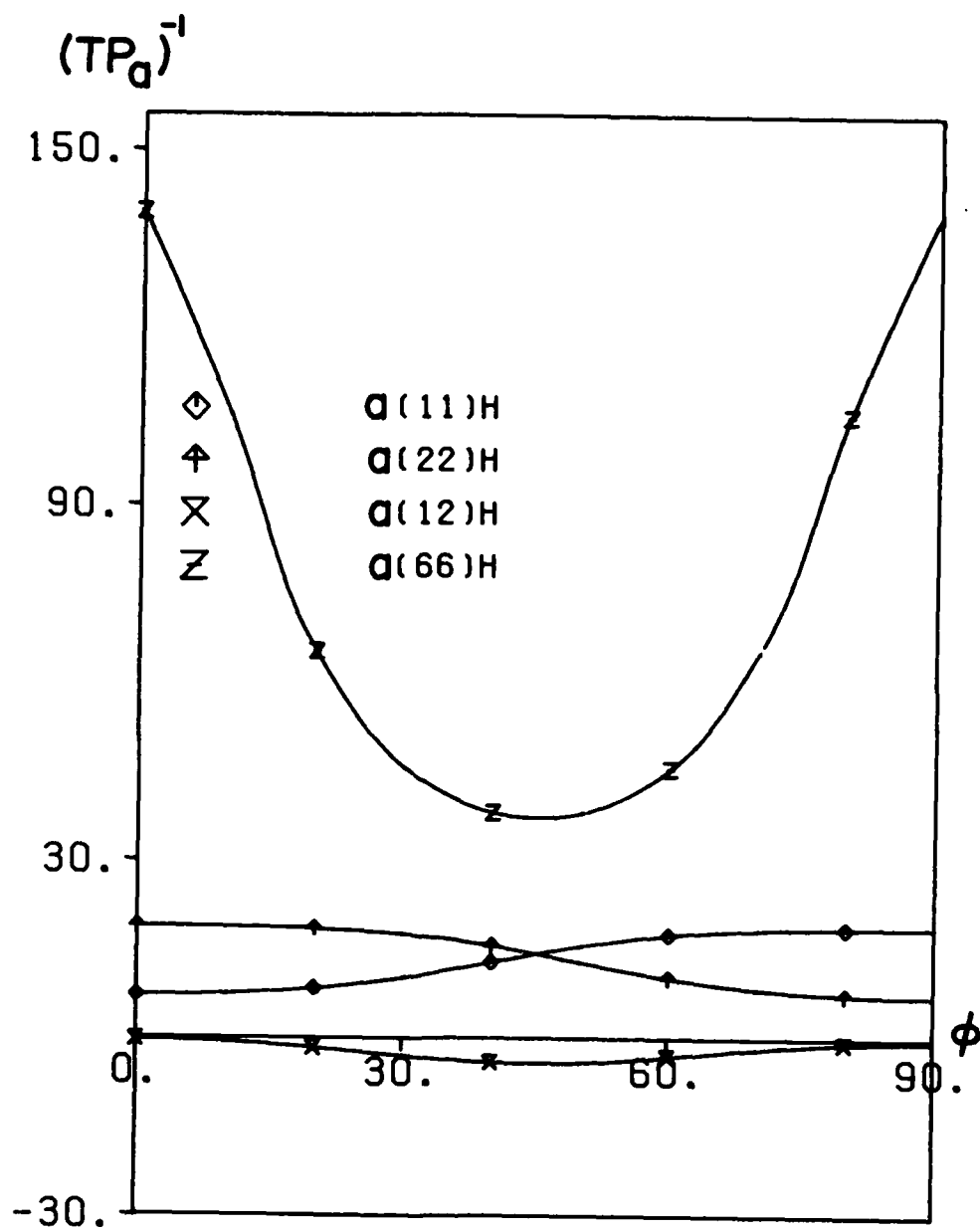
FIGURE 12 COMPLIANCE OF LAMINATE D
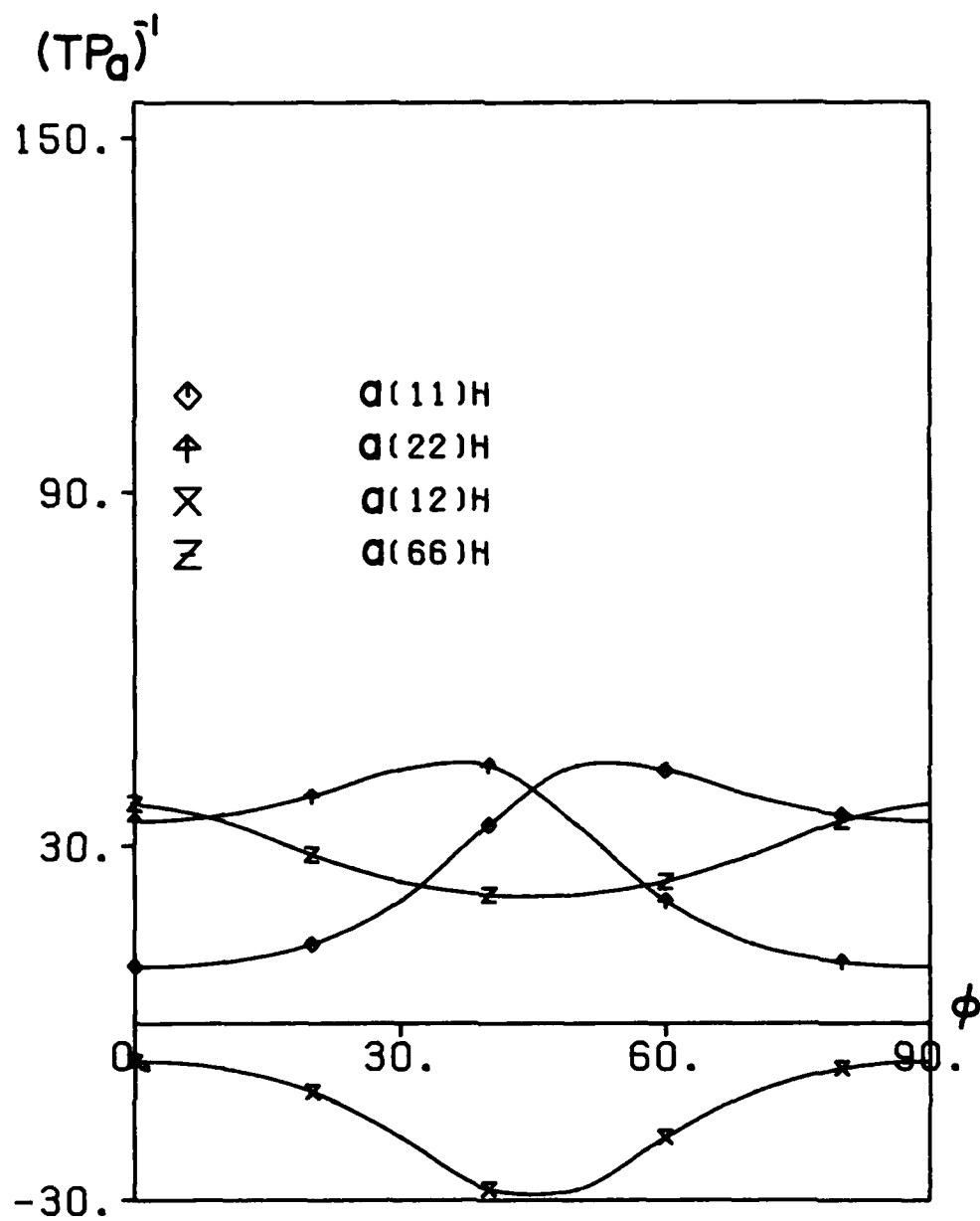
FIGURE 13 COMPLIANCE OF LAMINATE E
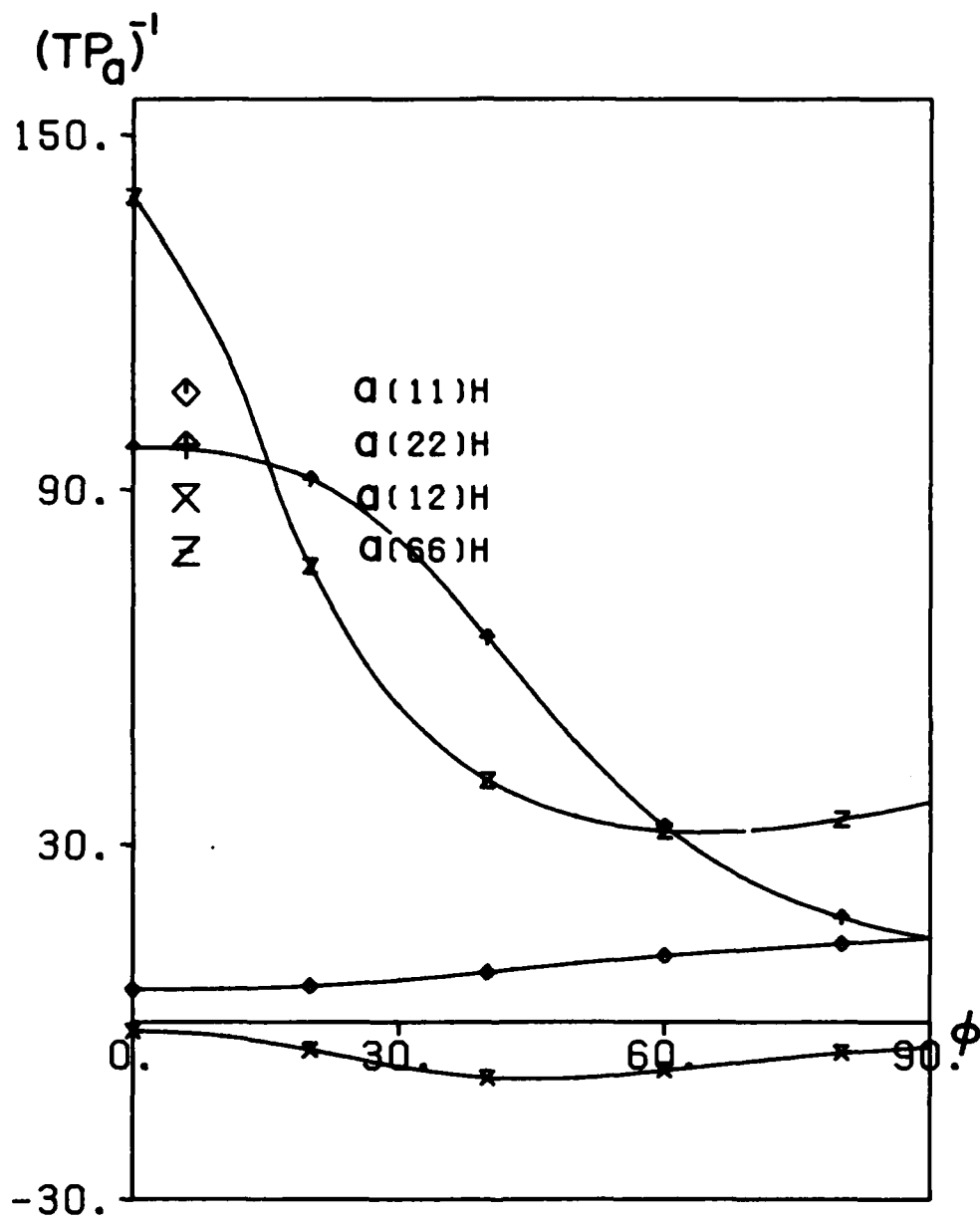
FIGURE 14 COMPLIANCE OF LAMINATE F
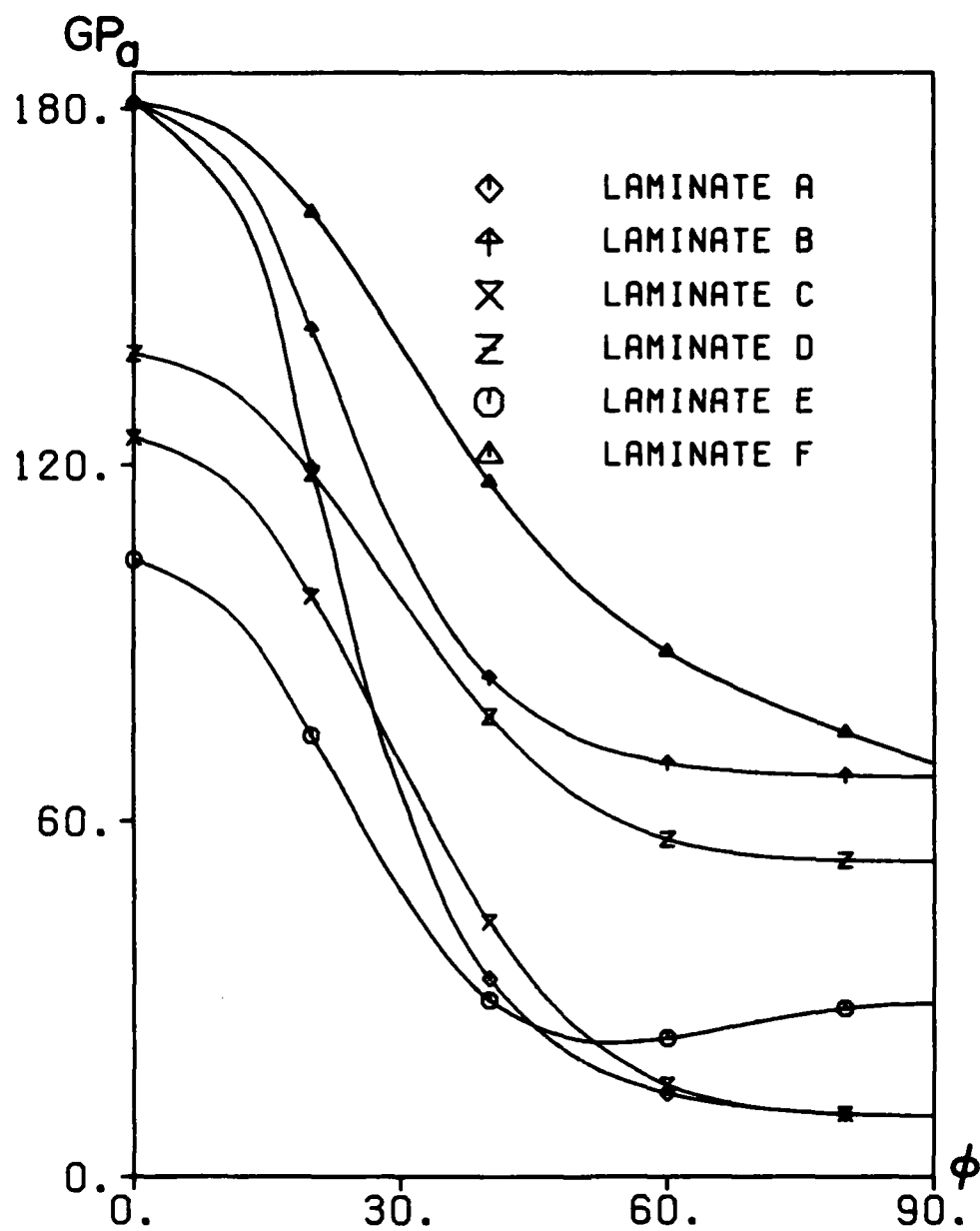
FIGURE 15 ENGINEERING CONSTANT E_1^o
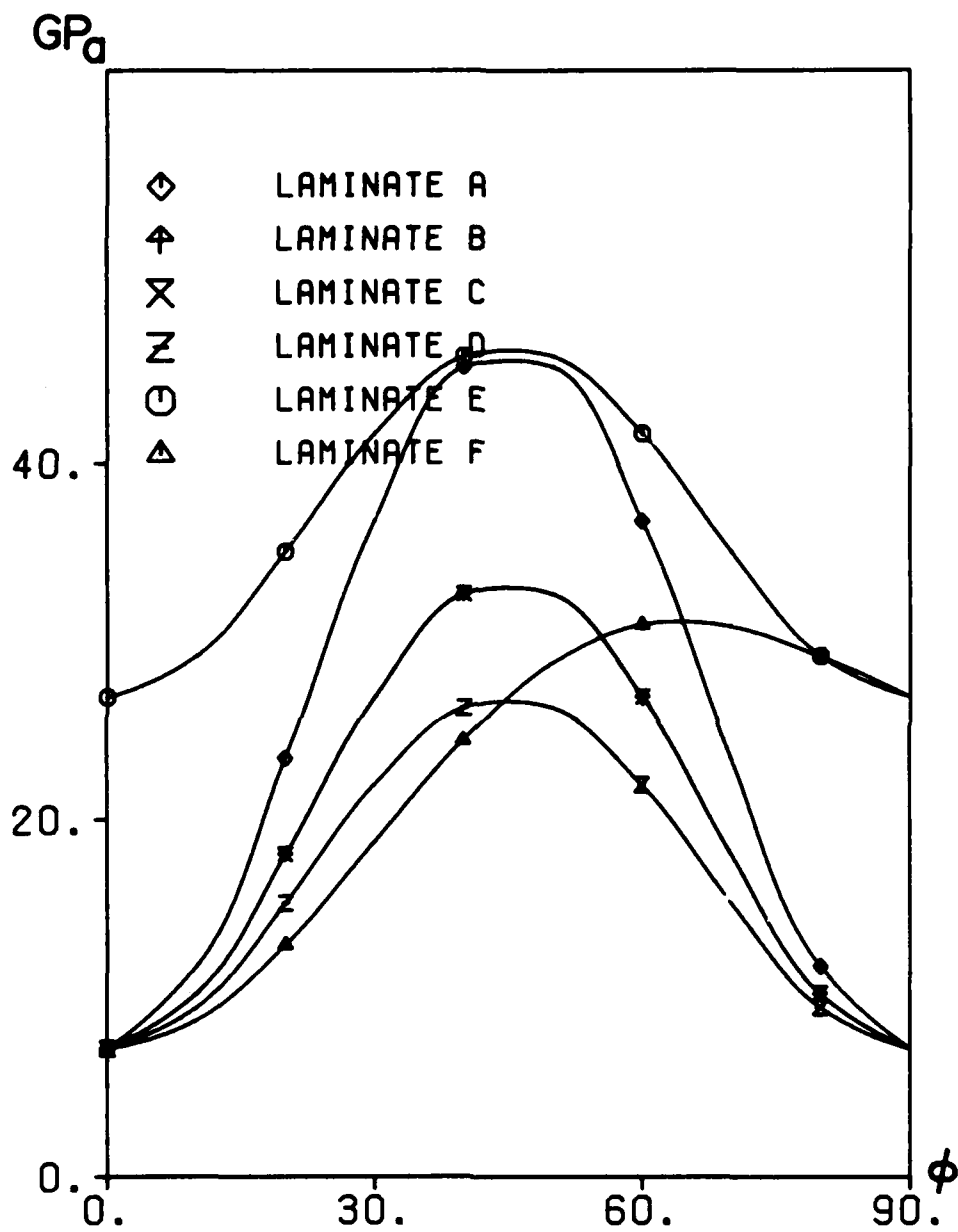
FIGURE 16 ENGINEERING CONSTANT E_6^o
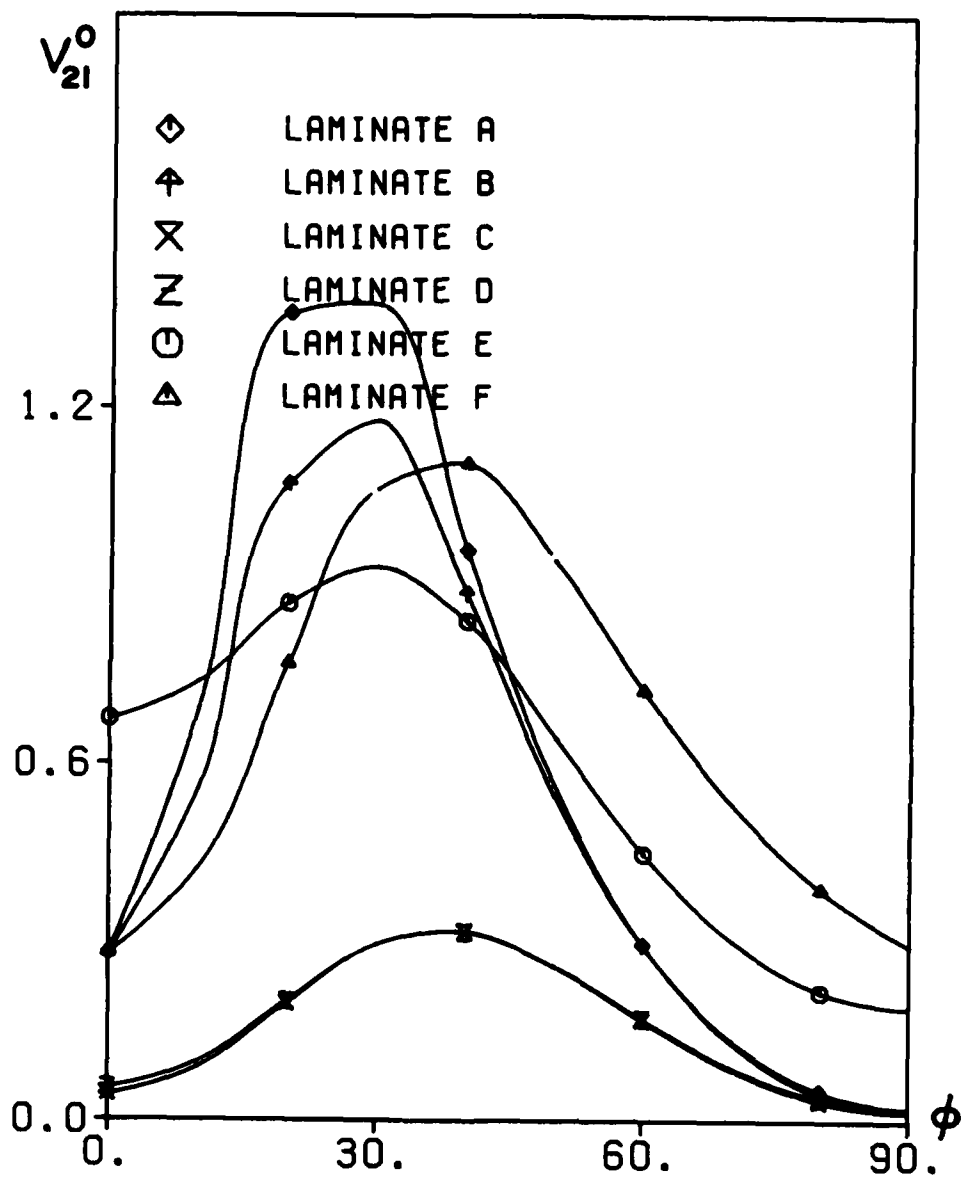
FIGURE 17 ENGINEERING CONSTANT V_{21}^0

FIGURE 18   LAMINATE A : $\{-\phi,\phi\}$

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FIGURE 19 LAMINATE A: $\{-\phi, \phi\}$

$N_1 = 1$

$N_1 = 0.8$

$N_1 = 0.6$

$N_1 = 0.4$

$N_1 = 0.2$

$N_1 = 0$

$N_6 = 1$

29
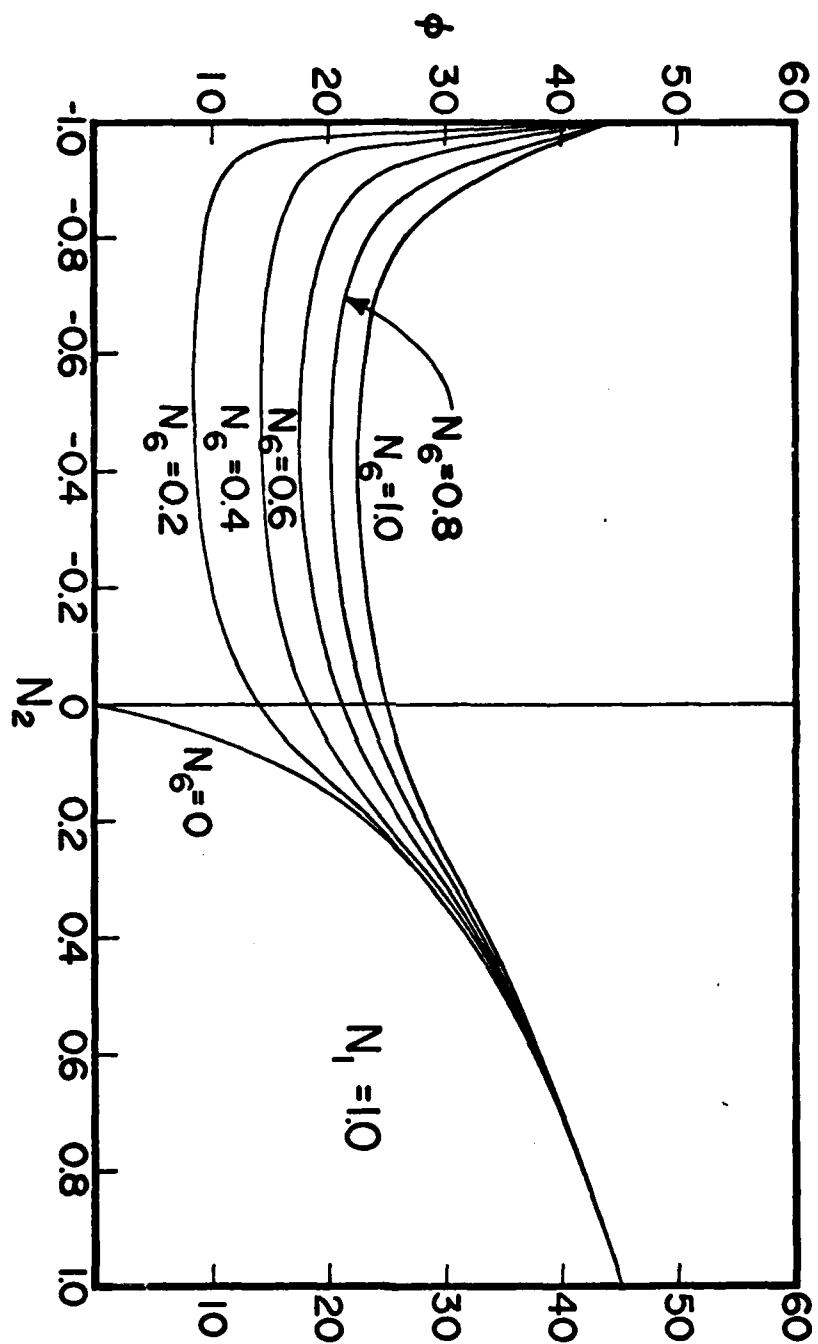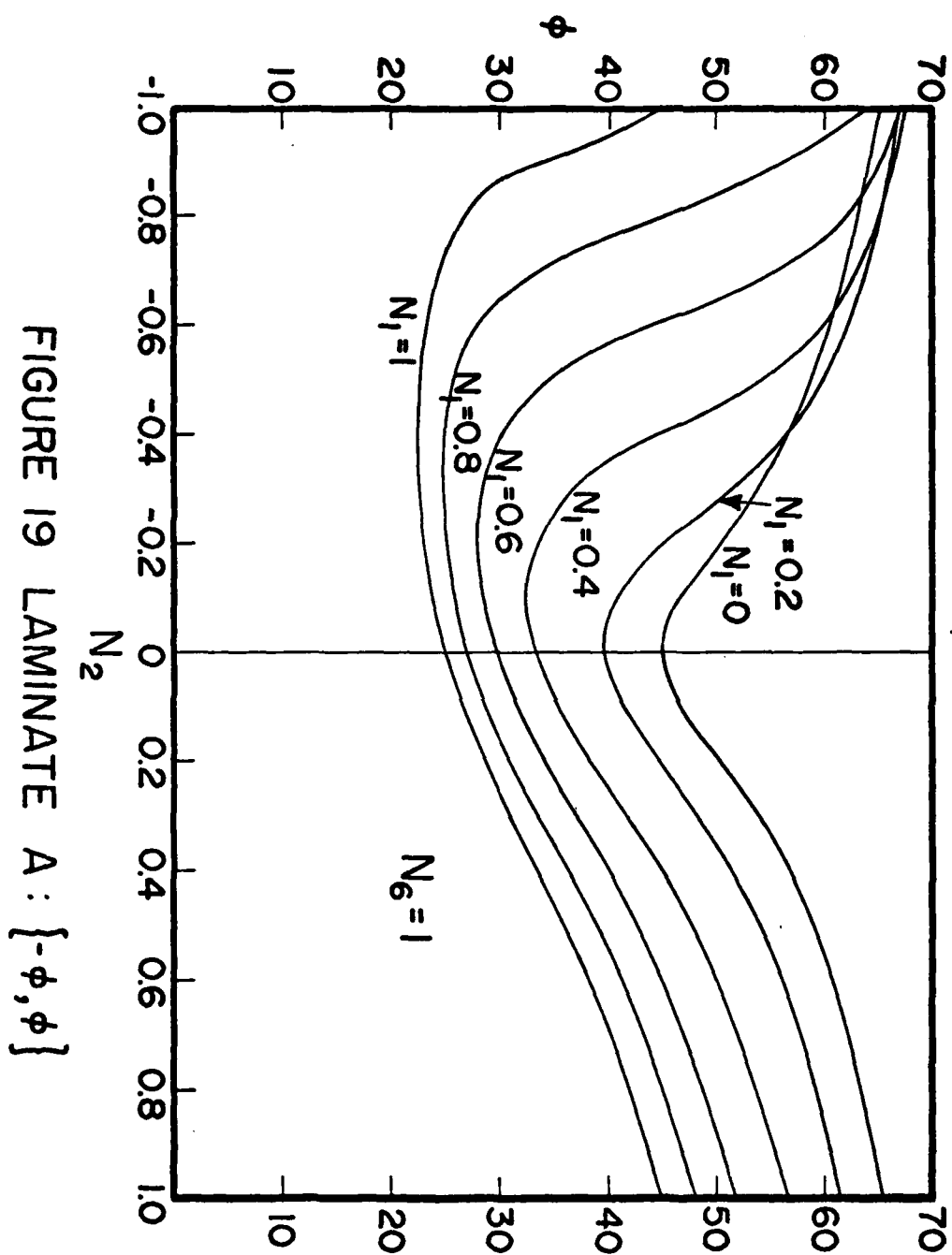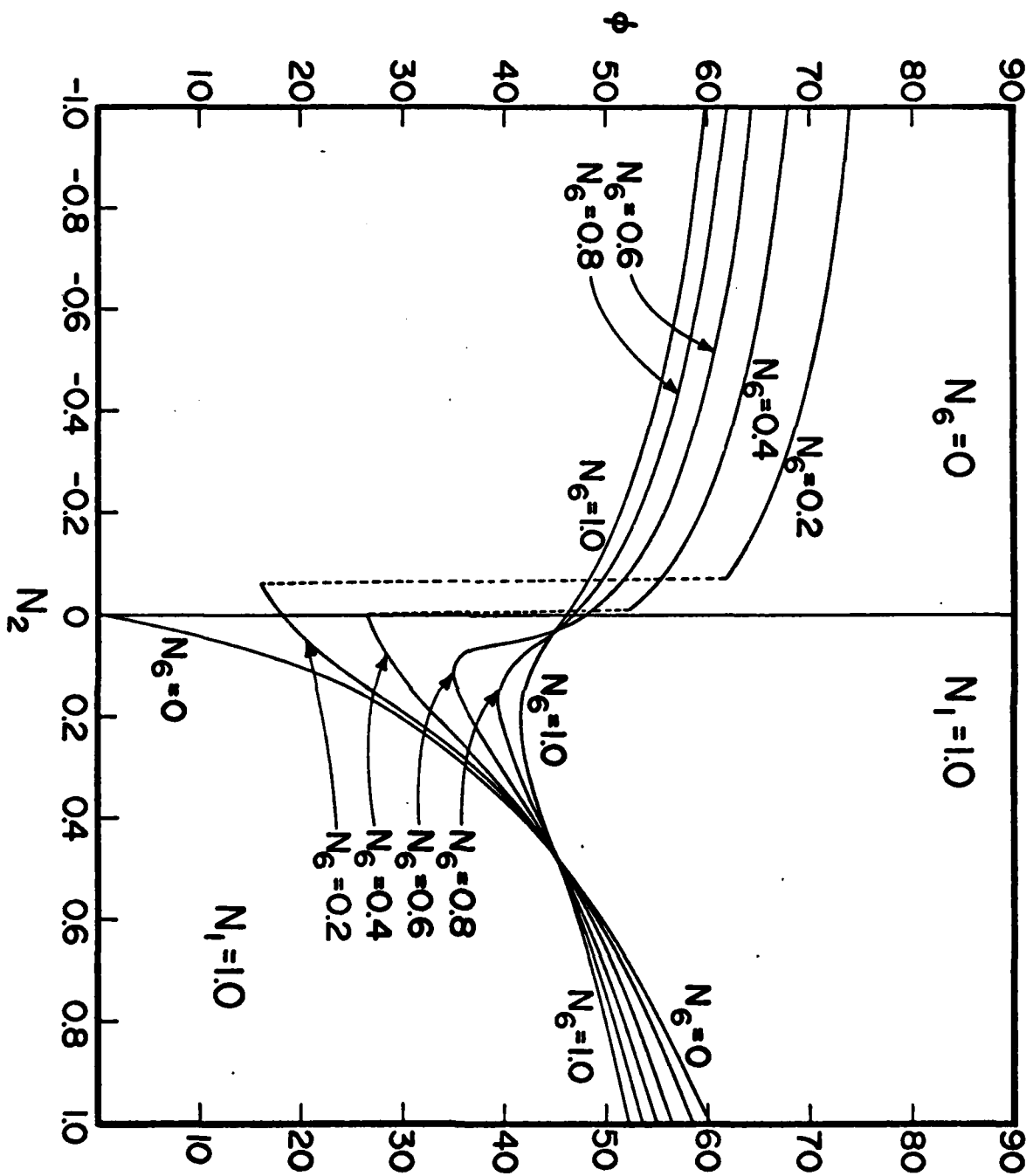
FIGURE 20   LAMINATE B : [0, -φ, φ]
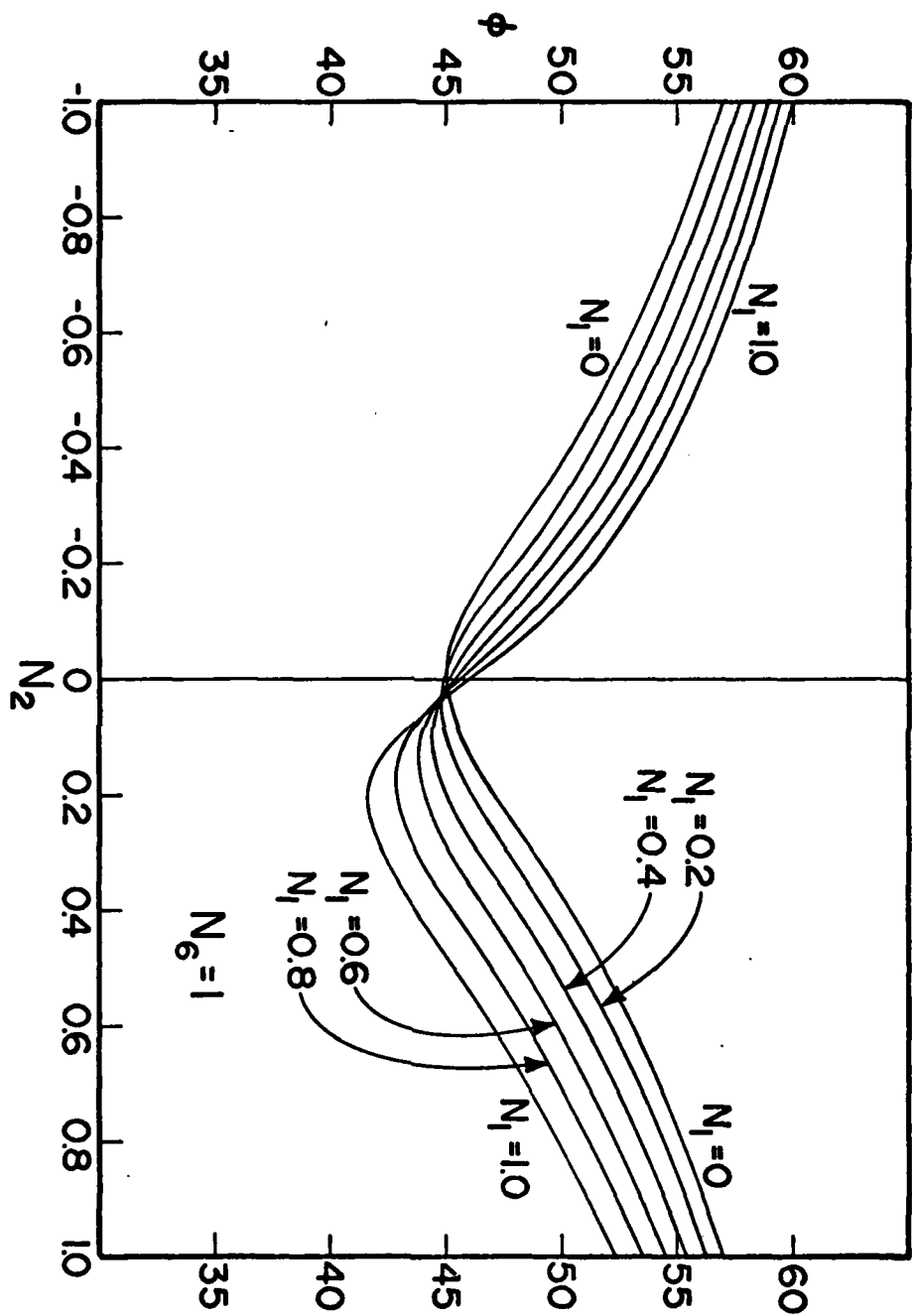
FIGURE 21 LAMINATE B: $\{0, -\phi, \phi\}$
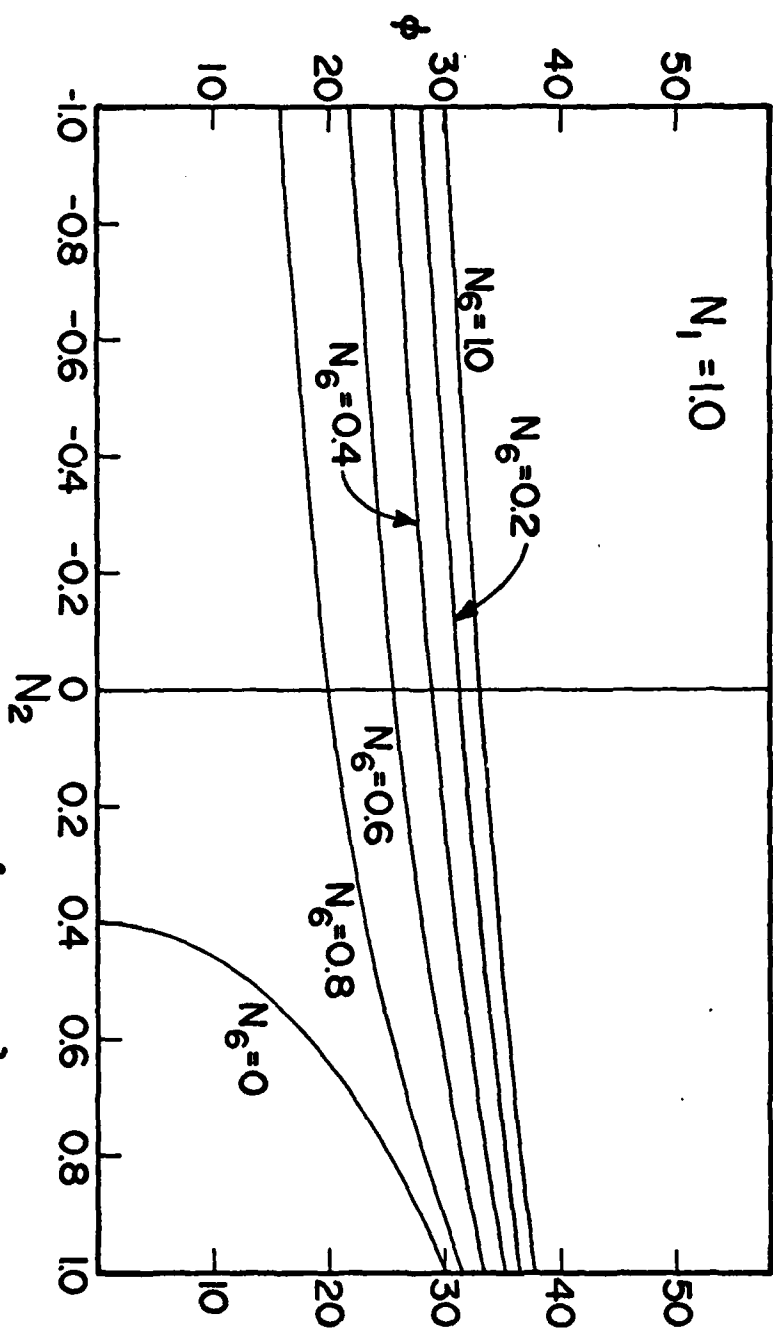
FIGURE 22 LAMINATE C: {90,-φ,φ}
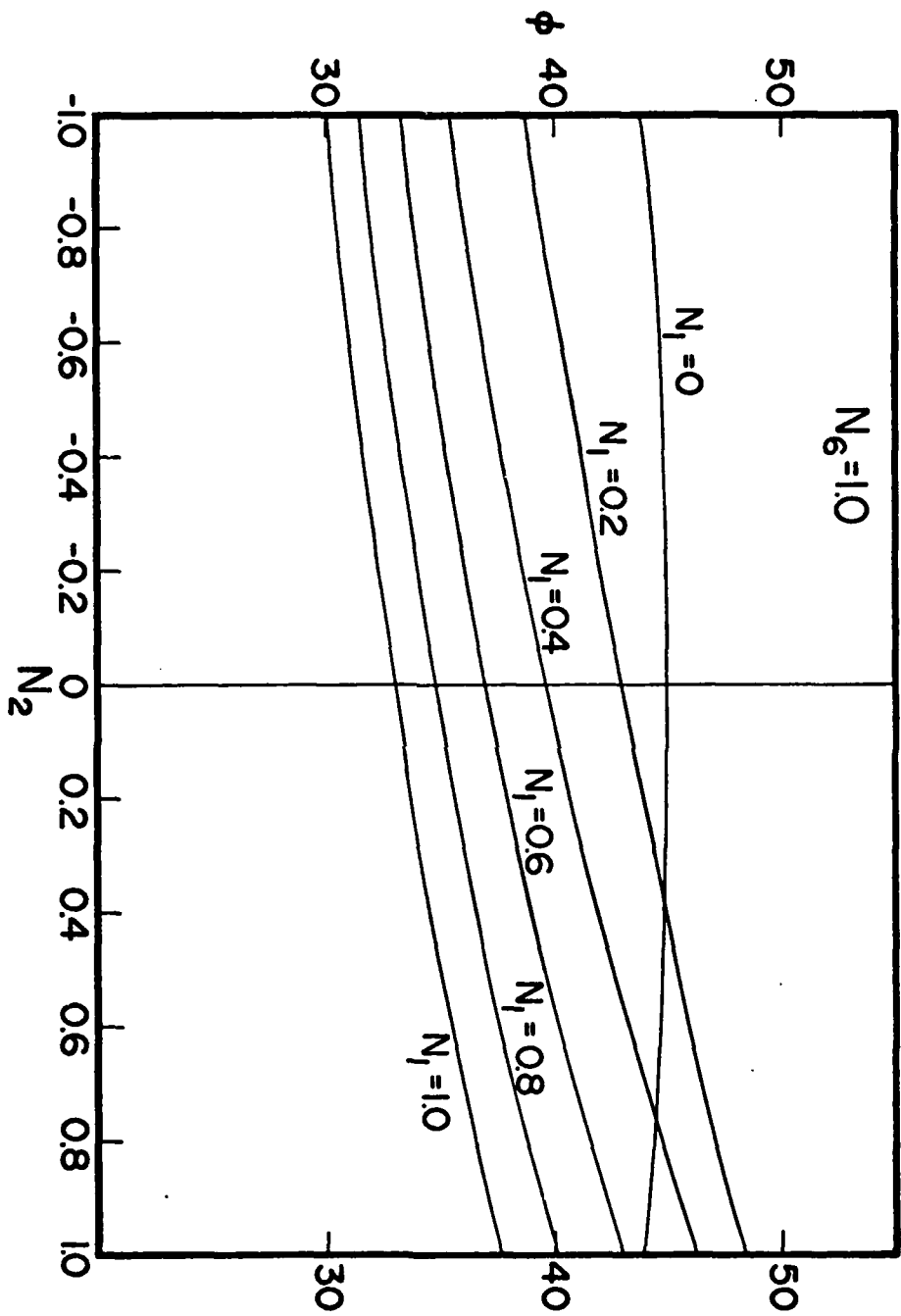
FIGURE 23 LAMINATE C: {90,-φ,φ}
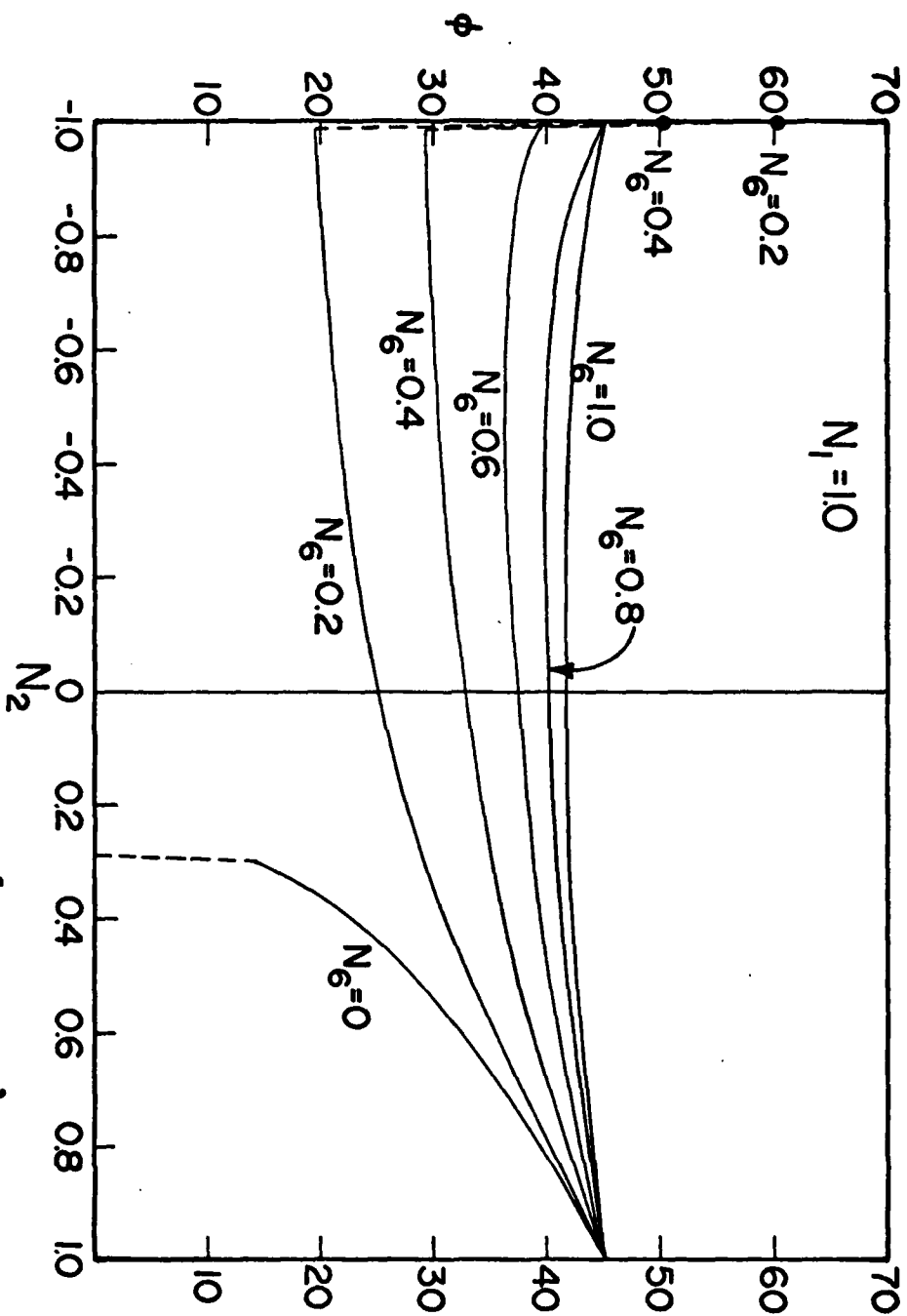
FIGURE 24 LAMINATE D : {0,90,-φ,φ}
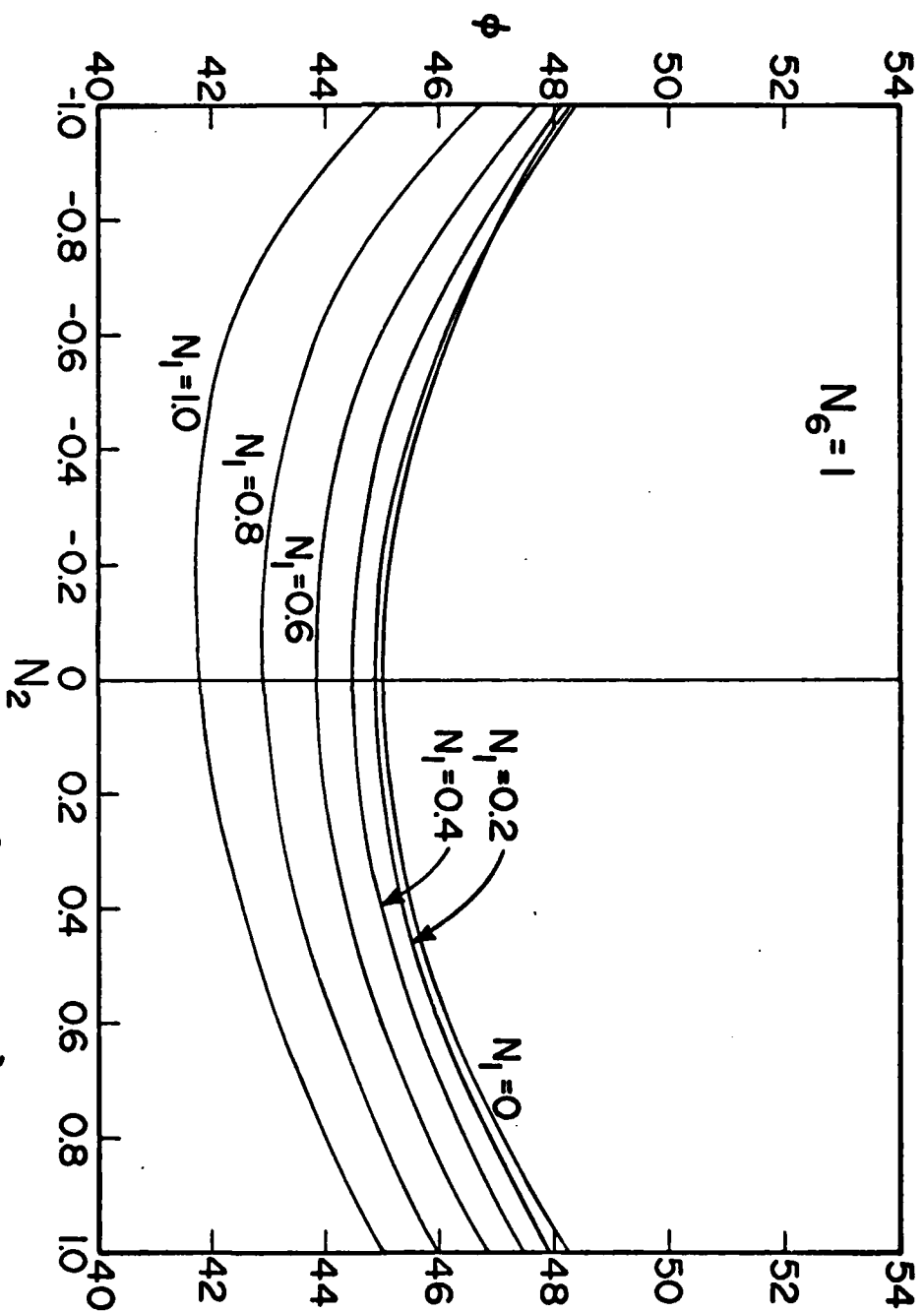
FIGURE 25 LAMINATE :D {0,90,-φ,φ}
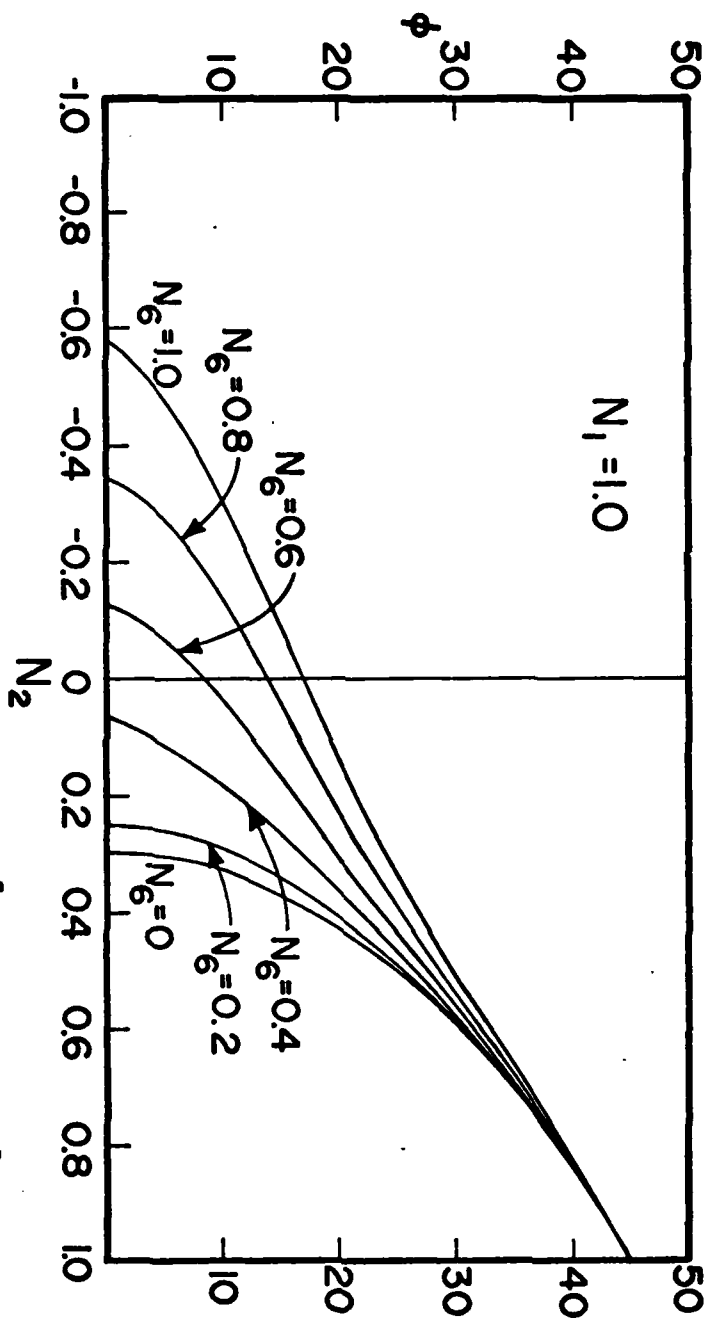
FIGURE 26   LAMINATE E : $\{-45,45,-\phi,\phi\}$

36
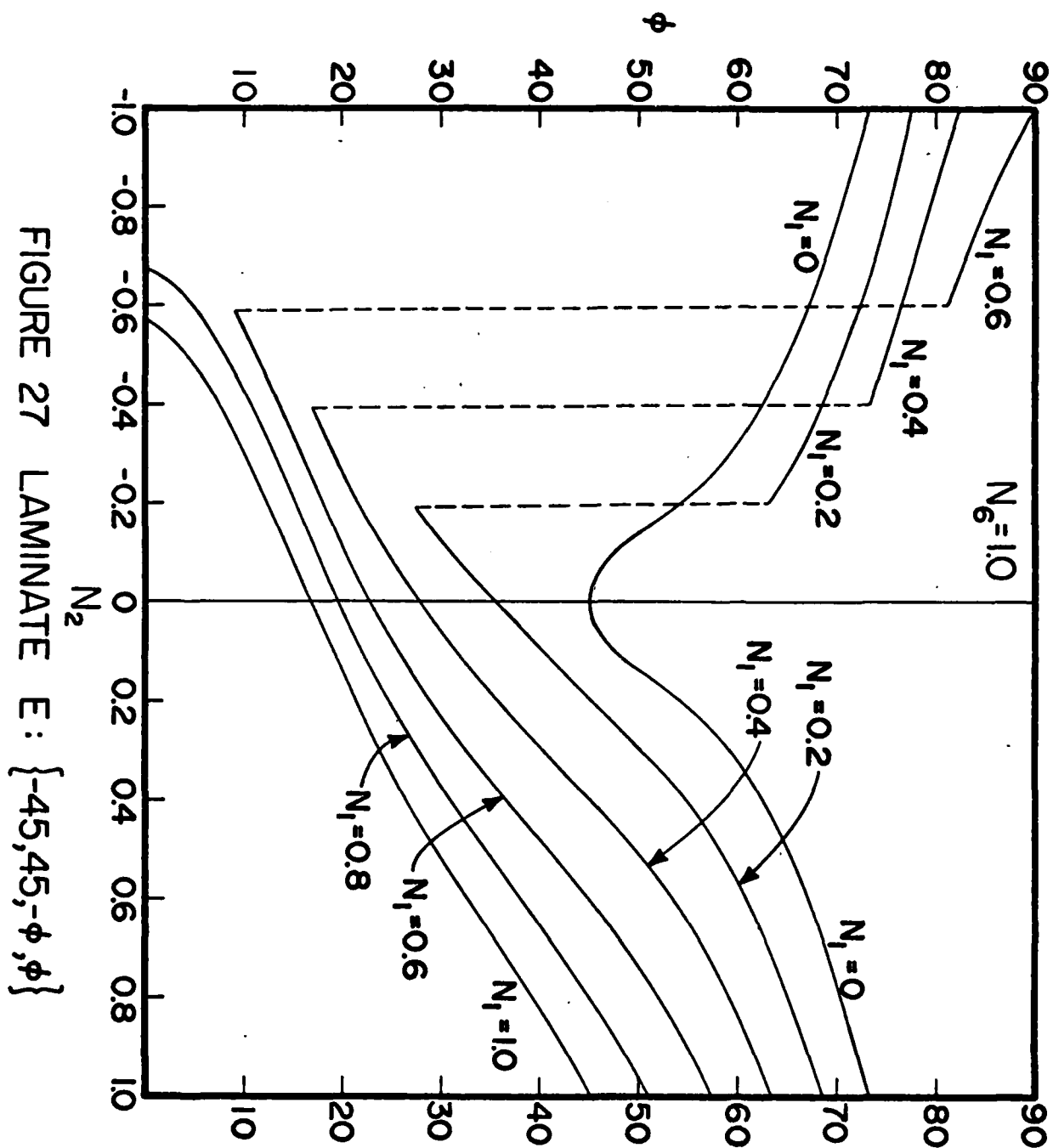
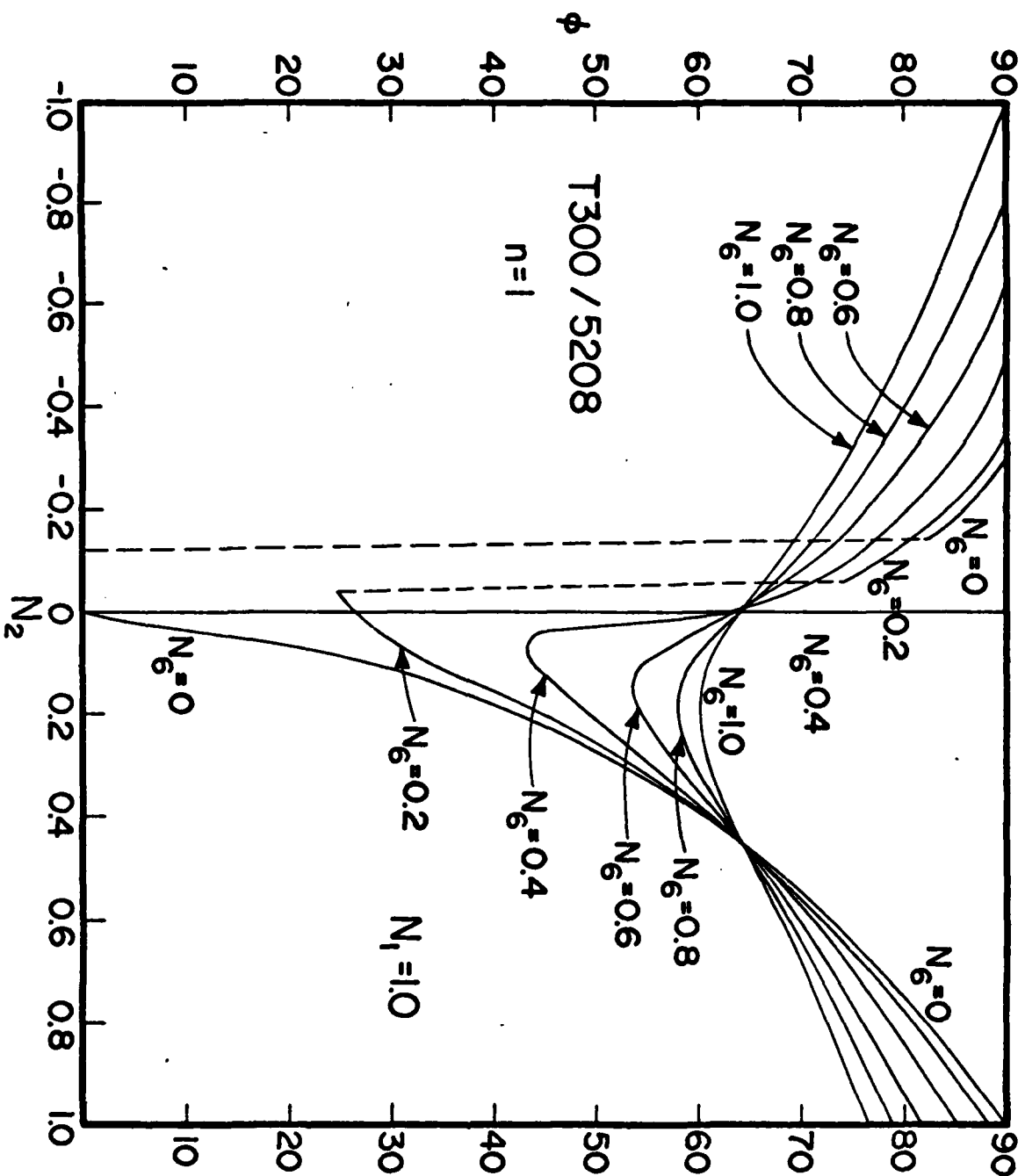FIGURE 27 LAMINATE E: $\{-45, 45, -\phi, \phi\}$

FIGURE 28 LAMINATE: CONTINUOUS LAMINATE

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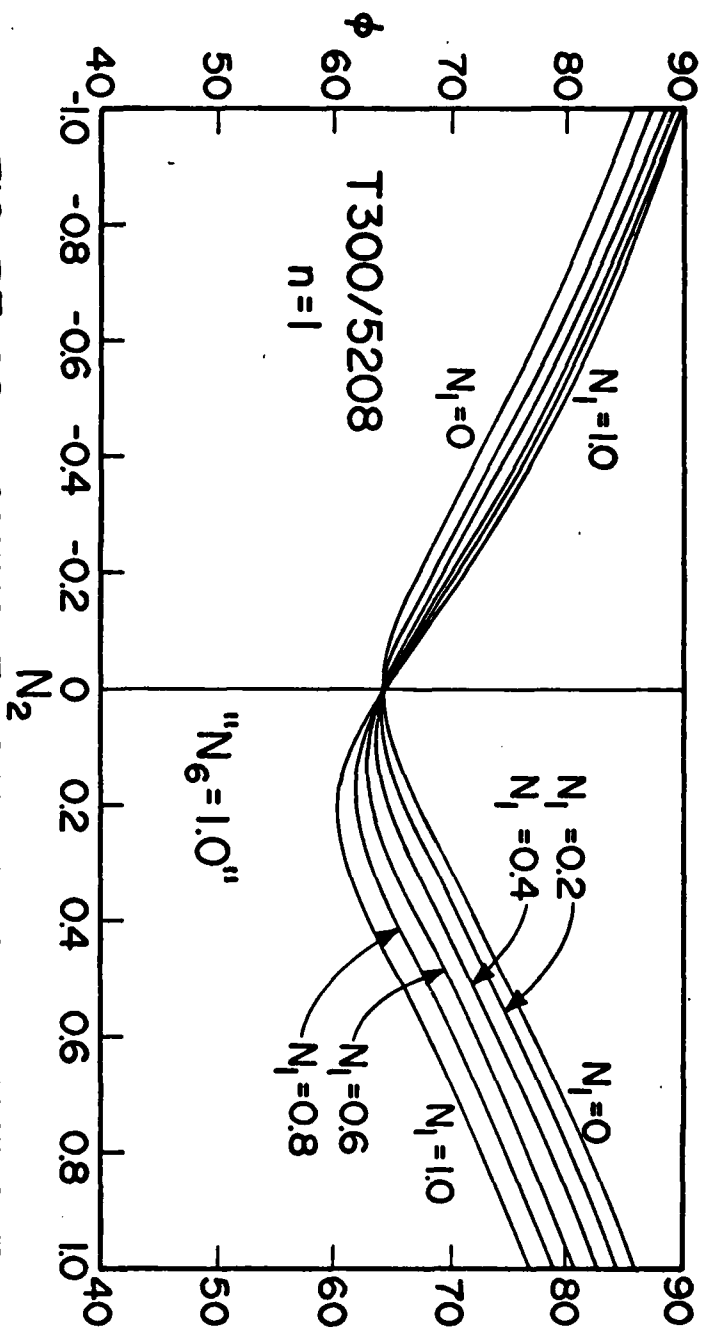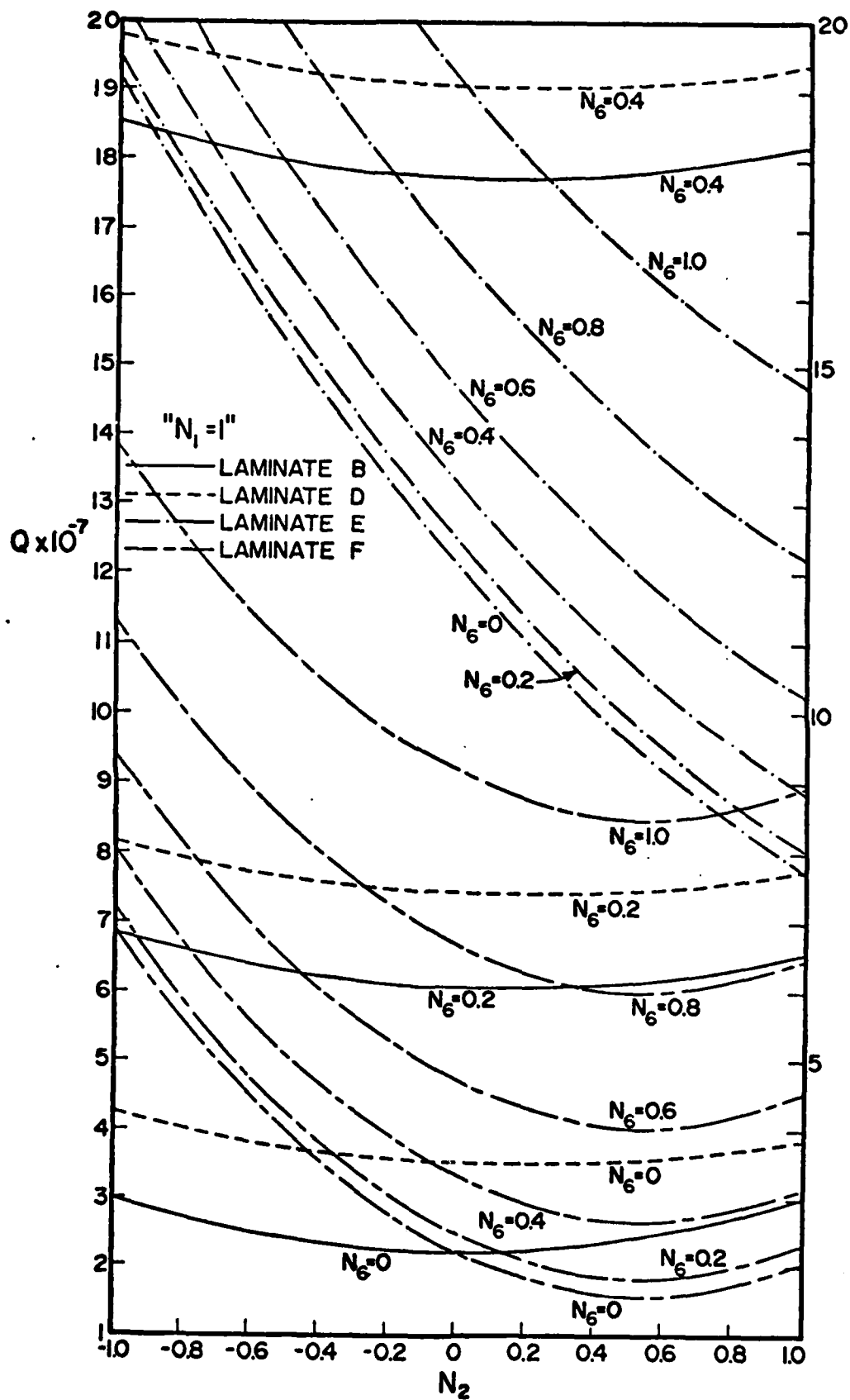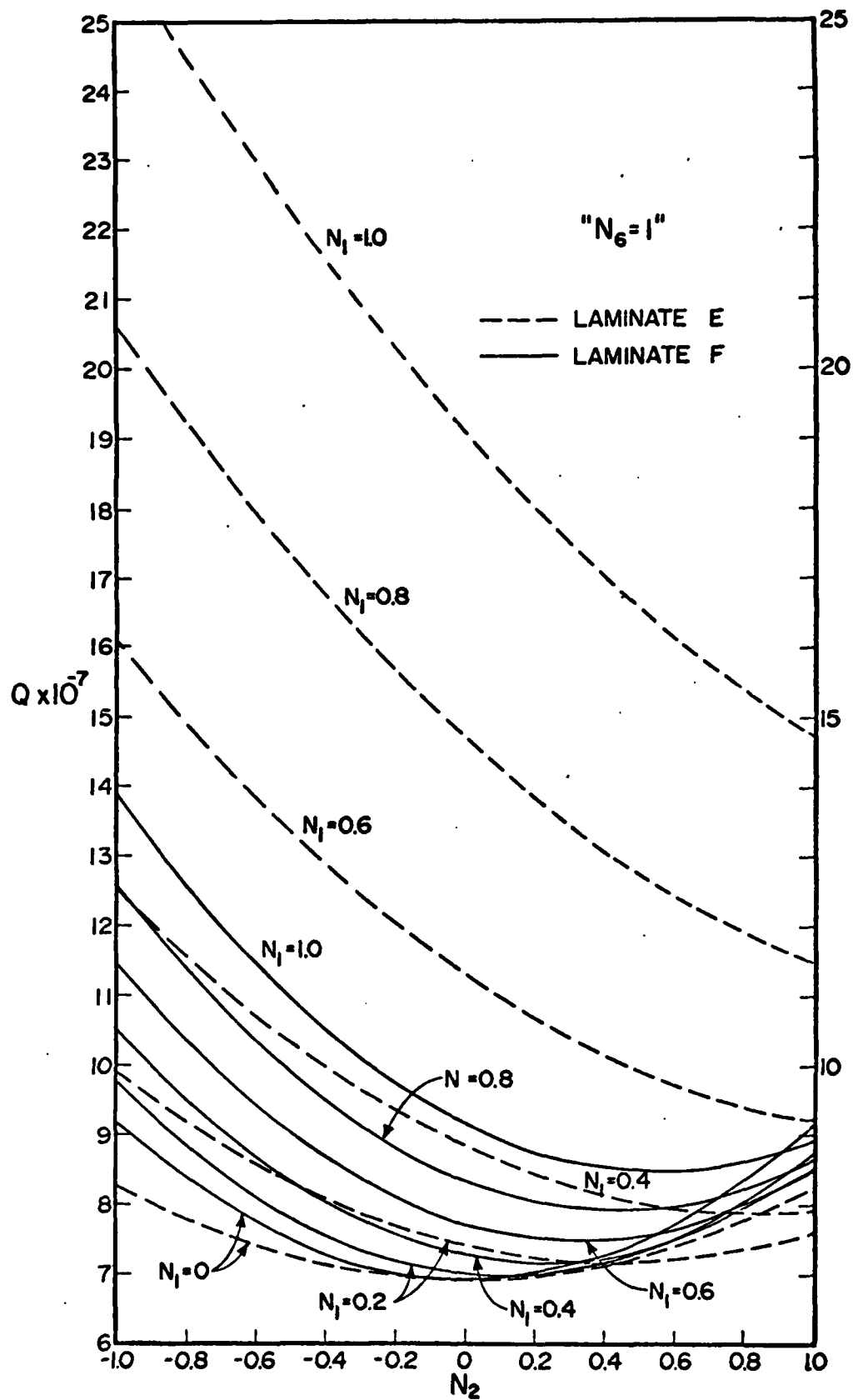
FIGURE 29 LAMINATE : CONTINUOUS LAMINATE

FIGURE 30 COMPARISON OF Q VALUES

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FIGURE 31 COMPARISON OF Q VALUES